List of formulas for Math 113, Fall 2012

- Distance formula: $|P_1P_2| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$
- Equation of a sphere with center $C(h,k,l)$ and radius r

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2.$$

- The **length** of a vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}.$$

- If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then the **dot product** of $\mathbf{a}$ and $\mathbf{b}$ is the number

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3.$$

- If θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$, then

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$$

- The **direction cosines** of the the vector $\mathbf{a}$:

$$\cos\alpha = \frac{a_1}{|\mathbf{a}|},\ \cos\beta = \frac{a_2}{|\mathbf{a}|},\ \cos\gamma = \frac{a_3}{|\mathbf{a}|}.$$

- Scalar projection of $\mathbf{b}$ onto $\mathbf{a}$:

$$\mathrm{comp}_a\mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$$

- Vector projection of $\mathbf{b}$ onto $\mathbf{a}$

$$\mathrm{proj}_a b = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2}\mathbf{a}$$

- The work done by a constant force $\mathbf{F}$ is

$$\mathbf{F} \cdot \mathbf{D}.$$

- $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then the **cross product** of $\mathbf{a}$ and $\mathbf{b}$ is the vector

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

- The volume of a parallelepiped determined by the vectors $\mathbf{a}, \mathbf{b}$, and $\mathbf{c}$ is the magnitude of their scalar triple product:

$$V = |a \cdot (b \times c)|$$

- If v is a vector parallel to L and let r_0 is the position vector of P_0, the **vector equation** of the line L through P is

$$\mathbf{r} = \mathbf{r_0} + t\mathbf{v},$$

where t is the **parameter.**

- The **parametric equations** of a line**:**

$$\begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \\ z &= z_0 + ct \end{aligned}$$

- The line segment from $\mathbf{r}_0$ and $\mathbf{r}_1$ is given by the vector equation

$$\mathbf{r}(t) = (1-t)\mathbf{r}_0 + t\mathbf{r}_1 \quad 0 \leq t \leq 1$$

- A plane is determined by a point $P_0(x_0, y_0, z_0)$ in the plane and a vector $\mathbf{n}$ that is orthogonal to the plane.

$$\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$$

- The **scalar equation** of the plane through $P_0(x_0, y_0, z_0)$ with normal vector $n = \langle a, b, c \rangle$ is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0.$$

- The **linear equation** of a plane

$$ax + by + cz + d = 0$$

- If two planes are not parallel, then they intersect in a straight line and the angle between them is the (acute) angle between their normal vectors.

- If $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$, then

$$\lim_{t \to a} \mathbf{r}(t) = \langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \rangle$$

- If $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$, where f, g, and h are differentiable functions, then
$$\mathbf{r}'(t) = \langle f'(t), g'(t), h'(t) \rangle$$

- The vector $\mathbf{r}'(t)$ is called the **tangent vector**.

- The **unit tangent vector** is
$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$$

- The **length** of a space curve $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$ is
$$\begin{aligned} L &= \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2 + [h'(t)]^2} \mathrm{d}t \\ &= \int_a^b \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}z}{\mathrm{d}t}\right)^2} \mathrm{d}t \\ &= \int_a^b |r'(t)| dt \end{aligned}$$

- The **arc length function** s is
$$s(t) = \int_a^t |r'(u)| du = \int_a^t \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2 + \left(\frac{dz}{du}\right)^2} du.$$

- The **curvature** of a curve is
$$k = \left| \frac{d\mathbf{T}}{ds} \right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|r'(t)|^3}.$$

- Let $\mathbf{r}(t)$ be a space curve. The **velocity vector** $\mathbf{v}(t)$ at time t is
$$\mathbf{v}(t) = \lim_{h \to 0} \frac{\mathbf{r}(t+h) - \mathbf{r}(t)}{h} = \mathbf{r}^{'}(t).$$

- The **speed** of the particle at time t is the magnitude of the velocity, that is, $|\mathbf{v}(t)|$.

- The **acceleration** of the particle is defined as the derivative of the velocity:
$$\mathbf{a}(t) = \mathbf{v}^{'}(t) = \mathbf{r}^{''}(t).$$