LIST OF FORMULAS FOR MATH 113, FALL 2012, PART 3

- The **double integral** of f over the rectangle R is
$$\iint_R f(x,y)dA = \lim_{m,n\to\infty} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A.$$
- A **Riemann sum** of f over the rectangle R is $\sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A$.
- **Midpoint Rule:**
$$\iint_r f(x,y)dA \simeq \sum_{i=1}^{m}\sum_{j=1}^{n} f(\overline{x}_i, \overline{y}_j)\Delta A,$$
where $\overline{x}_i$ is the midpoint of $[x_{i-1}, x_i]$ and $\overline{y}_j$ is the midpoint of $[y_{j-1}, y_j]$.
- Let f be a function of two variables that is integrable on the rectangle $R = [a,b] \times [c,d]$. There are two **iterated integrals**
$$\int_a^b \int_c^d f(x,y)dydx$$
and
$$\int_c^d \int_a^b f(x,y)dxdy.$$
- A domain D is of **type I** if it lies between the graphs of two continuous functions of x:
$$D = \{(x,y) \,|\, a \le x \le b,\ g_1(x) \le y \le g_2(x)\}.$$
- If D is a region of type I and f is continuous then
$$\iint_D f(x,y)dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)dydx.$$
- A domain D is of **type II** if it can be expressed as
$$D = \{(x,y) \,:\, c \le y \le d,\ h_1(y) \le x \le h_2(y)\}.$$
- If D is a region of type II and f is continuous then
$$\iint_D f(x,y)dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)dxdy.$$
- $\iint_D 1dA =$ area of $D = A(D)$.
- If $m \le f(x,y) \le M$, then $mA(D) \le \iint_D f(x,y)dA \le MA(D)$.
- The **polar coordinates** (r,θ) of a point are related to the rectangular coordinates (x,y) by the equations
$$\begin{aligned} r^2 &= x^2 + y^2 \\ x &= r\cos\theta \\ y &= r\sin\theta. \end{aligned}$$

- A **polar rectangle** is a domain that can be expressed as
$$D = \{(r, \theta) \,|\, a \leq r \leq b, \alpha \leq \theta \leq \beta\}.$$
- If f is continuous on a polar rectangle R given by $0 \leq a \leq r \leq b$, $\alpha \leq \theta \leq \beta$, where $0 \leq \beta - \alpha \leq 2\pi$, then
$$\iint_R f(x, y) dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta) r dr d\theta.$$
- If f is continuous on a polar region of the form
$$D = \{(r, \theta) \,:\, \alpha \leq \theta \leq \beta,\, h_1(\theta) \leq r \leq h_2(\theta)\}$$
then
$$\iint_D f(x, y) dA = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta, r\sin\theta) r dr d\theta.$$
- If f is continuous on the rectangular box $B = [a, b] \times [c, d] \times [r, s]$, then
$$\iiint_B f(x, y, z) dV = \int_r^s \int_c^d \int_a^b f(x, y, z) dx dy dz.$$
We can change the order of integration in the above formula.