HOMEWORK 1

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Problem 1 (Exercise 1.2.1 on page 11).

(1) Prove that $\sqrt{3}$ is irrational. Does a similar argument work to show $\sqrt{6}$ is irrational?
(2) Where does the proof of Theorem 1.1.1 break down if we try to use it to prove that $\sqrt{4}$ is irrational?

Problem 2 (Exercise 1.5.4 on page 31). Let S be the set consisting of all sequences of 0's and 1's. Observe that S is not a particular sequence, but rather a large set whose elements are sequences, namely

$$S = \{(a_1, a_2, a_3, \dots) : a_n = 0 \text{ or } 1\}$$

Problem 3 (Exercise 1.3.3 a) on page 17). Let A be bounded below, and define $B = \{b \in \mathbb{R} : b \text{ is a lower bound for } A\}$. Show that $\sup B = \inf A$.

Problem 4 (Exercise 1.3.4 on page 18). Assume that A and B are nonempty, bounded above, and satisfy $B \subseteq A$. Show that $\sup B \leq \sup A$.

Problem 5 (Exercise 1.3.6 in page 18). Compute, without proofs, the suprema and infima of the following sets:

(a) $\{n \in \mathbb{N} : n^2 < 10\}$.
(b) $\{n/(m+n) : m, n \in \mathbb{N}\}$.
(c) $\{n/(2n+1) : n \in \mathbb{N}\}$.
(d) $\{n/m : m, n \in \mathbb{N} \text{ with } m + n \leq 10\}$.

Problem 6 (Exercise 1.3.7 on page 18). Prove that if a is an upper bound for A, and if a is also an element of A, then it must be that $a = \sup A$.