Problem 1 (Exercise 6.2.3 on page 161). Consider the sequence of functions

\[ h_n(x) = \frac{x}{1 + x^n} \]

over the domain \([0, \infty)\).

(a) Find the pointwise limit of \((h_n)\) on \([0, \infty)\).
(b) Explain how we know that the convergence cannot be uniform on \([0, \infty)\).
(c) Choose a smaller set over which the convergence is uniform and supply an argument to show that this is indeed the case.

Problem 2 (Exercise 6.2.7 on page 161). Assume that \((f_n)\) converges uniformly to \(f\) on \(A\) and that each \(f_n\) is uniformly continuous. Prove that \(f\) is uniformly continuous on \(A\).

Problem 3 (Exercise 6.3.1 on page 166). (a) Let

\[ h_n(x) = \frac{\sin(nx)}{n} \]

Show that \(h_n \to 0\) uniformly on \(\mathbb{R}\). At what points does the sequence of derivatives \(h'_n\) converge?
(b) Modify this example to show that it is possible for a sequence \((f_n)\) to converge uniformly but for \((f'_n)\) to be unbounded.

Problem 4 (Exercise 6.4.3 on page 168). (a) Show that \(g(x) = \sum_{n=1}^{\infty} \cos(2^n x)/2^n\) is continuous on all of \(\mathbb{R}\).
(b) Prove that \(h(x) = \sum_{n=1}^{\infty} x^n/n^2\) is continuous on \([-1, 1]\).

Problem 5 (Exercise 6.5.1 on page 174). Consider the function \(g\) defined by the power series

\[ g(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots \]

Date: Due Sunday, 12/09/2012; You do not have to type this assignment using Latex.
(a) Is \( g \) define on \((-1, 1)\)? Is it continuous on this set? Is \( g \) defined on \((-1, 1]\)? Is it continuous on this set? What happens on \([-1, 1]\)? Can the power series for \( g(x) \) possible converge for any other points \( |x| > 1 \)? Explain.

(b) For what values of \( x \) is \( g'(x) \) defined? Find a formula for \( g' \).

**Problem 6** (Exercise 6.5.2 on page 174). Find suitable coefficients \( (a_n) \) so that the resulting power series \( \sum a_n x^n \)

(a) converges absolutely for all \( x \in [-1, 1] \) and diverges off of this set;

(b) converges conditionally at \( x = -1 \) and diverges at \( x = 1 \);

(c) converges conditionally at both \( x = -1 \) and \( x = 1 \).

(d) Is it possible to find an example of a power series that converges conditionally at \( x = -1 \) and converges absolutely at \( x = 1 \)?