Problem 1 (Exercise 2.3.1 on page 49). Show that the constant sequence \((a, a, a, a, \ldots)\) converges to \(a\).

Problem 2 (Exercise 2.3.2 on page 49). Let \(x_n \geq 0\) for all \(n \in \mathbb{N}\)

(a) If \((x_n) \to 0\), show that \((\sqrt{x_n}) \to 0\).

(b) If \((x_n) \to x\), show that \((\sqrt{x_n}) \to \sqrt{x}\).

Problem 3 (Exercise 2.3.3 on page 49, Squeeze Theorem). Show that if \(x_n \leq y_n \leq z_n\) for all \(n \in \mathbb{N}\), and if \(\lim x_n = \lim z_n = \ell\), then \(\lim y_n = \ell\) as well.

Problem 4 (Exercise 2.3.6 on page 49). (a) Show that if \((b_n) \to b\), then the sequence of absolute values \(|b_n|\) converges to \(|b|\).

(b) Is the converse of part (a) true? If we know that \(|b_n| \to |b|\), can we deduce that \((b_n) \to b\)?

Problem 5 (Exercise 2.3.8 in page 50). Give an example of each of the following, or state that such a request is impossible by referencing the proper theorem(s).

(a) sequences \((x_n)\) and \((y_n)\), which both diverge, but whose sum \((x_n + y_n)\) converges;

(b) sequences \((x_n)\) and \((y_n)\), when \((x_n)\) converges \((y_n)\) diverges, and \((x_n + y_n)\) converges;

(c) a convergent sequence \((b_n)\) with \(b_n \neq 0\) for all \(n\) such that \((1/b_n)\) diverges;

(d) an unbounded sequence \((a_n)\) and a convergent sequence \((b_n)\) with \(a_n - b_n\) bounded;

(e) two sequences \((a_n)\) and \((b_n)\), where \((a_nb_n)\) and \(a_n\) converge but \((b_n)\) does not.

Problem 6 (Exercise 2.3.9 on page 50). Does Theorem 2.3.4 remain true if all of the inequalities are assumed to be strict? If we assume, for instance, that a convergent sequence \((x_n)\) satisfies \(x_n > 0\) for all \(n \in \mathbb{N}\), what may we conclude about the limit?

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