HOMEWORK #3, REAL ANALYSIS I, FALL 2012

MARIUS IONESCU

Problem 1 (Exercise 2.3.1 on page 49)**.** Show that the constant sequence $(a, a, a, a, \ldots)$ converges to a.

Problem 2 (Exercise 2.3.2 on page 49)**.** Let $x_n \geq 0$ for all $n \in \mathbb{N}$

(a) If $(x_n) \to 0$, show that $(\sqrt{x_n}) \to 0$.
(b) If $(x_n) \to x$, show that $(\sqrt{x_n}) \to \sqrt{x}$.

Problem 3 (Exercise 2.3.3 on page 49, **Squeeze Theorem**)**.** Show that if $x_n \leq y_n \leq z_n$ for all $n \in \mathbb{N}$, and if $\lim x_n = \lim z_n = \ell$, then $\lim y_n = \ell$ as well.

Problem 4 (Exercise 2.3.6 on page 49)**.** (a) Show that if $(b_n) \to b$, then the sequence of absolute values $|b_n|$ converges to $|b|$.
(b) Is the converse of part (a) true? If we know that $|b_n| \to |b|$, can we deduce that $(b_n) \to b$?

Problem 5 (Exercise 2.3.8 in page 50)**.** Give an example of each of the following, or state that such a request is impossible by referencing the proper theorem(s).

(a) sequences (x_n) and (y_n), which both diverge, but whose sum $(x_n + y_n)$ converges;
(b) sequences (x_n) and (y_n), whenr (x_n) converges (y_n)diverges, and $(x_n + y_n)$ converges;
(c) a convergent sequence (b_n) with $b_n \neq 0$ for all n such that $(1/b_n)$ diverges;
(d) an unbounded sequence (a_n) and a convergent sequence (b_n) with $a_n - b_n)$ bounded;
(e) two sequences (a_n) and (b_n), where $(a_n b_n)$ and a_n converge but (b_n) does not.

Problem 6 (Exercise 2.3.9 on page 50)**.** Does Theorem 2.3.4 remain true if all of the inequalities are assumed to be strict? If we assume, for instance, that a convergent sequence (x_n) satisfies $x_n > 0$ for all $n \in \mathbb{N}$, what may we conclude about the limit?

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