HOMEWORK #6, REAL ANALYSIS I, FALL 2012

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Problem 1 (Exercise 3.2.2 on page 83)**.** Let

$$B = \left\{ \frac{(-1)^n n}{n+1} : n = 1, 2, 3, \ldots \right\} .$$

(a) Find the limit points of B.
(b) Is B a closed set?
(c) Is B an open set?
(d) Find $\overline{B}$.

Problem 2 (Exercise 3.2.3 on page 83)**.** Decide whether the following sets are open, closed, or neither. If a set is not open, find a point in the set for which there is no ε-neighborhood contained in the set. If a set is not closed, find a limit point that is not contained in the set.

(a) $\mathbb{Q}$.
(b) $\mathbb{N}$.
(c) $\{x \in \mathbb{R} : x > 0\}$.
(d) $(0, 1] = \{x \in \mathbb{R} : 0 < x \leq 1\}$.
(e) $\{1 + 1/4 + 1/9 + \cdots + 1/n^2 : n \in \mathbb{N}\}$.

Problem 3 (Exercise 3.2.7 on page 83)**.** Let $x \in O$, where O is an open set. If (x_n) is a sequence converging to x, prove that all but a finite number of the terms of (x_n) must be contained in O.

Problem 4 (Exercise 3.2.8 on page 83)**.** Given $A \subseteq \mathbb{R}$, let L be the set of all limit points of A.

(a) Show that the set L is closed.
(b) Argue that if x is a limit point of $A \cup L$, then x is a limit point of A. Use this observation to furnish a proof for Theorem 3.2.12.

Problem 5. (a) If y is a limit point of $A \cup B$, show that y is either a limit point of A or a limit point of B (or both).
(b) Prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.
(c) Does the result about closures in (b) extend to infinite unions of sets?

Date: **Due Friday, 10/26/2012.**

Problem 6. Decide whether the following statements are true or false. Provide counterexamples for those that are false, and supply proofs for those that are true.

(a) For any set $A \subseteq \mathbb{R}$, $\overline{A}^c$ is open.
(b) If a set A has an isolated point, it cannot be an open set.
(c) A set A is closed if and only if $\overline{A} = A$.
(d) If A is a bounded set, then $s = \sup A$ is a limit point of A.
(e) Every finite set is closed.
(f) An open set that contains every rational number must necessarily be all of $\mathbb{R}$.