HOMEWORK #7, REAL ANALYSIS I, FALL 2012

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Problem 1 (Exercise 3.3.1 on page 87)**.** Show that if K is compact, then $\sup K$ and $\inf K$ both exist and are elements of K.

Problem 2 (Exercise 3.3.3 on page 88)**.** Show that the Cantor set defined in Section 3.1 is a compact set.

Problem 3 (Exercise 3.3.5 on page 88)**.** Decide which of the following sets are compact. For those that are not compact, show how Definition 3.3.1 breaks down. In other words, give an example of a sequence contained in the given set that does not possess a subsequence converging to a limit in the set.

(a) $\mathbb{Q}$.
(b) $\mathbb{Q} \cap [0, 1]$.
(c) $\mathbb{R}$.
(d) $\mathbb{Z} \cap [1, 10]$.
(e) $\{1, 1/2, 1/3, 1/4, 1/5, \dots\}$.
(f) $\{1, 1/2, 2/3, 3/4, 4/5, \dots\}$.

Problem 4 (Exercise 4.2.1 on page 108)**.** Use Definition 4.2.1 to supply a proof for the following statements.

(a) $\lim_{x\to 2}(2x + 4) = 8$.
(b) $\lim_{x\to 0} x^3 = 0$.
(c) $\lim_{x\to 2} x^3 = 8$.
(d) $\big[[x]\big] = 3$, where $\big[[x]\big]$ denotes the greatest integer less than or equal to x.

Problem 5 (Exercise 4.2.2 on page 108)**.** Assume a particular $\delta > 0$ has been constructed as a suitable response to a particular ε challenge. Then, any larger/smaller (*pick one*) δ will also suffice. (**Please justify your answer**).

Problem 6 (Exercise 4.2.6 on page 108)**.** Let $g : A \to \mathbb{R}$ and assume that f is a bounded function on A (i.e., there exists $M > 0$ satisfying $|f(x)| \le M$ for all $x \in A$). Show that if $\lim_{x\to c} g(x) = 0$ then $\lim_{x\to c} g(x)f(x) = 0$.

Date: **Due Friday, 11/2/2012.**