Problem 1 (Exercise 5.2.2 on page 136). (a) Use Definition 5.2.1 to product the proper formula for the derivative of \( f(x) = 1/x \).
(b) Combine the result of (a) with the chain rule (Theorem 5.2.5) to supply a proof for part (iv) of Theorem 5.2.4 [the derivative rule for quotients].
(c) Supply a direct proof of Theorem 5.2.4 (iv) by algebraically manipulating the difference quotient for \((f/g)\) in a style similar to the proof of Theorem 5.2.4 (iii).

Problem 2 (Exercise 4.2.4 on page 136). Let \( f_a(x) = \begin{cases} x^a & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \).
(a) For which values of \( a \) is \( f \) continuous at zero?
(b) For which values of \( a \) is \( f \) differentiable at zero? In this case, is the derivative function continuous?
(c) For which values of \( a \) is \( f \) twice-differentiable?

Problem 3 (Exercise 5.3.1 on page 143). Recall from Exercise 4.4.9 that a function \( f : A \to \mathbb{R} \) is “Lipschitz on \( A \)” if there exists an \( M > 0 \) such that
\[
\left| \frac{f(x) - f(y)}{x - y} \right| \leq M
\]
for all \( x, y \in A \). Show that if \( f \) is differentiable on a closed interval \([a, b]\) and if \( f' \) is continuous on \([a, b]\), then \( f \) is Lipschitz on \([a, b]\).

Problem 4 (Exercise 5.3.5 on page 143). A fixed point of a function \( f \) is a value \( x \) where \( f(x) = x \). Show that if \( f \) is differentiable on an interval with \( f'(x) \neq 1 \), then \( f \) can have at most one fixed point.

Problem 5 (Exercise 5.3.7 on page 143). Recall that a function \( f : (a, b) \to \mathbb{R} \) is increasing on \((a, b)\) if \( f(x) \leq f(y) \) whenever \( x \leq y \) in \((a, b)\). Assume \( f \) is differentiable on \((a, b)\). Show that \( f \) is increasing on \((a, b)\) if and only if \( f'(x) \geq 0 \) for all \( x \in (a, b)\).

Problem 6 (Exercise 6.2.1 on page 160). Let
\[
f_n(x) = \frac{nx}{1 + nx^2}.
\]
(a) Find the pointwise limit of \((f_n)\) for all \(x \in (0, \infty)\).
(b) Is the convergence uniform on \((0, \infty)\)?
(c) Is the convergence uniform on \((0, 1)\)?
(d) Is the convergence uniform on \((1, \infty)\)?