MATH 300, IN CLASS PROBLEMS
WEEK 1

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Problem 1. If (a_n) is a sequence of real numbers, then in any closed interval of $\mathbb{R}$ with positive length there exists a real number x such that $x \neq a_n$ for each n.

Problem 2. Let f be a strictly increasing mapping of $\mathbb{R}$ onto $\mathbb{R}^+$ such that $f(0) = 1$ and $f(x + y) = f(x)f(y)$. Prove that $f(x) = a^x$, where $a = f(1) > 1$.

Problem 3. Let (a_n) be a sequence in $\mathbb{R}$. Prove that if the three subsequences (a_{2n}), (a_{2n+1}), and (a_{3n}) are convergent, then so is (a_n).

Problem 4. What is the last digit of 3^{4798}?

Problem 5. You have a well-shuffled deck of 52 playing cards. You ask a friend to divide the deck into 3 piles. You then be that one of the top 3 cards is a face card (Jack, Queen, or King). Who has a better chance of winning, you or your friend?

Problem 6. Let f be a real-valued function on the plane such that for every square $ABCD$ in the plane, $f(A) + f(B) + f(C) + f(D) = 0$. Does it follow that $f(P) = 0$ for all points P in the plane?