MATH 399, HOMEWORK #6

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From the Sierpinski gasket notes: problems 3.1.2 (on page 68), 3.2.4 and 3.2.8 (on page 72).

Problem 4. Prove that if the terms of the Harmonic Series containing a 7 in (the base-10 numerals for) their denominator are deleted, the resulting series converges.

Problem 5. Find the Fourier series representation of the saw-toothed function $f(x)$ defined by:

$$f(x) = \begin{cases} x + \pi \text{ for } x \in (-\pi, 0] \\ \pi - x \text{ for } x \in (0, \pi] \end{cases},$$

and $f(x + 2\pi) = f(x)$ for all x. Use your answer to evaluate $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

Problem 6. A little boy has his dollar changed into 6 coins. Running down the street he drops one coin into a sewer grate. What is the probability that it was a dime?

Problem 7. Let S be a set of 52 distinct positive integers. Prove that there exist $x, y \in S$ such that either $x + y$ or $|x - y|$ is divisible by 100.

Problem 8. Find a nonzero polynomial $P(x, y)$ such that $P(\lfloor a \rfloor, \lfloor 2a \rfloor) = 0$ for all real numbers a. (Note: $\lfloor \nu \rfloor$ is the greatest integer less than or equal to ν.)

Date: Due Friday, 03/11/2011.