The following is a list of common mistakes.

1. $$\frac{1}{a+b} \neq \frac{1}{a} + \frac{1}{b}$$

   Just try $a = b = 1$.

   $$\frac{1}{2} = \frac{1}{a+b} \neq \frac{1}{a} + \frac{1}{b} = 2$$

   Thus,

   $$\frac{1}{x^2 + x + \sqrt{x}} \neq x^{-2} + x^{-1} + x^{-1/2}$$

   To see that this is not true, just try $x = 1$.

2. $$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$$

   It goes without saying that

   $$\sqrt{a^2 + b^2} \neq a + b$$

   People make this mistake by writing

   $$\sqrt{x^2 + 1} = x + 1,$$

   but this is not equal. Again, just let $x = 1$.

   Similarly,

   $$\sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$$

3. Example of choosing incorrectly for integration by parts. Consider

   $$\int \frac{x}{\left(x^2+3\right)^2}\, dx$$

   Suppose that you are trying to do this with integration by parts. Some people choose $u = x$ and $dv = \left(x^2 + 3\right)^2\, dx$. This is incorrect. You should instead choose $u = x$ and $dv = \frac{1}{(x^2+3)^2}\, dx$.

   Another example:

   $$\int \frac{dx}{x^2 + 3 + \sqrt{x+3}}$$

   People would choose $u = x^2 + 3$ and $dv = \sqrt{x+3}\, dx$. This is incorrect since $x^2 + 3$ and $\sqrt{x+3}$ are not even multiplied together. Furthermore, they are both on the denominator.

4. Notation issue:

   For the integral

   $$\int \frac{1+e^x}{1-e^x}\, dx$$

   Let $u = e^x$ so that $du = e^x\, dx$ and $\frac{du}{e^x} = dx$. We have

   $$\int \frac{1+u}{1-u} \frac{du}{e^x} = \int \frac{1+u}{1-u} \frac{du}{u}.$$

   Notice the integral above on the left has both x's and u's. This is somewhat nitpicking, but you should not have both x's and u's after you substitute. To avoid this,

   $$u = e^x \;\Rightarrow\; \ln u = x \;\Rightarrow\; \frac{dx}{u} = dx.$$

5. For simplifying differential equations:

   (a) $\ln(a+b) \neq \ln a + \ln b$. Just try $a = b = 1$. The natural log (ln) does not distribute over addition or subtraction.

(b) $\ln(a-b) \neq \dfrac{\ln a}{\ln b}$. Just try $a = 3$ and $b = 2$. You're confusing this with $\ln a - \ln b = \dfrac{\ln a}{\ln b}$.

(c) If you have something like $-e^a = b - c$ and you're solving for a, then you have to take care of the minus sign first before you apply the natural log to both sides:

$$-e^a = b - c \qquad \Rightarrow \qquad e^a = c - b \qquad \Rightarrow \qquad \ln(e^a) = \ln(c-b) \qquad \Rightarrow \qquad a = \ln(c-b)$$

This is because $\ln(-a) \neq -\ln a$.

6. Be careful with equal signs.

Let $y = 3x^2$ and suppose you need to calculate $1 + \left(\dfrac{dy}{dx}\right)^2$. Don't write

$$\frac{dy}{dx} = 6x = 36x^2 = 1 + 36x^2 = 1 + \left(\frac{dy}{dx}\right)^2.$$

Note that at the red $=$, the expressions are not equal. Although this may show what you are thinking, the expressions are not equal. You should instead write

$$\frac{dy}{dx} = 6x \qquad \Rightarrow \qquad \left(\frac{dy}{dx}\right)^2 = 36x^2 \qquad \Rightarrow \qquad 1 + \left(\frac{dy}{dx}\right)^2 = 1 + 36x^2$$

The $\Rightarrow$ can be read as "implies."

7. How to solve inequalities with absolute values. Consider $|x-2| < 6$.

First the *wrong way*:

$$|x-2| < 6 \qquad \Rightarrow \qquad |x| - 2 < 6 \qquad \Rightarrow \qquad |x| < 8 \qquad \Rightarrow \qquad -8 < x < 8$$

The red $\Rightarrow$ is telling us that $|x-2| \neq |x| - 2$. You cannot solve this problem this way. To see that this is not correct, notice that, for example, $|-7-2| = |-9| = 9 \not< 6$, where -7 is in the interval we found above.

Now the *correct way*:

$$|x-2| < 6 \qquad \Rightarrow \qquad -6 < x - 2 < 6 \qquad \Rightarrow \qquad -4 < x - 2 < 8.$$

8. Suppose you have the equation $xy = 2x^2 + c$, where c is a constant, and you want to solve for y. Then you would have

$$y = 2x + \frac{c}{x}.$$

THIS IS CORRECT.

Don't absorb the x into the constant and make a new constant; that is, you <u>don't</u> want

$$y = 2x + c_1 \qquad \text{where} \qquad c_1 = \frac{c}{x}.$$

THIS IS WRONG! Here, c_1 is technically not a constant at this point.

9. Factorials. Note that $(2n)! \neq 2!n!$. To show that this is not true, let $n = 4$.

$$\begin{aligned} (2 \cdot 4)! &= 8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40,320 \\ 2! \cdot 4! &= 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 48 \end{aligned}$$

So, we have

$$(2(n+1))! = (2n+2)! = (2n+1)(2n+1)(2n)!.$$

Also note that $(2(n+1))! \neq 2(n+1)n!$.

10. If you are differentiating $\sum_{n=0}^{\infty} (5x)^n$, then you should first write $\sum_{n=0}^{\infty} 5^n x^n$. Notice that

$$\frac{d}{dx}\left[\sum_{n=0}^{\infty} 5^n x^n\right] = \sum_{n=1}^{\infty} 5^n n x^{n-1}$$

and, by the chain rule,

$$\frac{d}{dx}\left[\sum_{n=0}^{\infty} (5x)^n\right] = \sum_{n=1}^{\infty} n\,(5x)^{n-1} \cdot 5 = \sum_{n=1}^{\infty} n 5^{n-1} x^{n-1} \cdot 5 = \sum_{n=1}^{\infty} 5^n n x^{n-1}$$

So, it is easier – and safer – if you first write $\sum_{n=0}^{\infty} 5^n x^n$ first.