Not every function is equal to its Taylor (or Maclaurin) series. Here are a few examples.

1. Let $f(x) = |x|$ and $a = 1$. We have $f(1) = 1$. Recall that

$$f'(x) = \begin{cases} 1 & \text{if } x > 1 \\ \text{undefined} & \text{if } x = 0 \\ -1 & \text{if } x < 1 \end{cases}$$

   so that $f'(1) = 1$. Note that $f^{(n)}(x) = 0$ for all $n > 1$. So our Taylor series is simply $1 + (x - 1) = x$, but this does not equal $f(x) = |x|$.

2. Consider the function

$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

   at $a = 0$. First note that as $x \to 0$, we have $-1/x^2 \to -\infty$ so that $e^{-1/x^2} \to 0$. This show that $f(x)$ is continuous. Now, observe that

$$\begin{aligned} f'(0) &= \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{e^{-1/h^2} - 0}{h} = \lim_{h\to 0} \frac{1/h}{e^{1/h^2}} \Rightarrow \frac{\infty}{\infty} \\ &\downarrow\text{Ⓛ} \\ &= \lim_{h\to 0} \frac{-\frac{1}{h^2}}{-\frac{2e^{1/h^2}}{h^3}} = \lim_{h\to 0} \frac{h}{2e^{1/h^2}} = 0 \end{aligned}$$

   With *lots!* of work[1], you can show that $f^{(n)}(0) = 0$ for all $n \geq 0$. It follows that the Taylor series around the origin is simply a sum of zeroes; so it is identically zero. This is clearly not equal to $f(x)$.

[1] See http://alpha.math.uga.edu/~mklipper/3100/F14/indepth-week14-end-f14.pdf