Critical Points

Warren Weckesser

Here we take a look at the critical points of a function.

```
> with(plots):
Warning, the name changecoords has been redefined
```

```
> f := x^3-3*x+y^3-3*y;
```

$$f := x^3 - 3\,x + y^3 - 3\,y$$

Compute the first derivatives.

```
> fx := diff(f,x);
```

$$fx := 3\,x^2 - 3$$

```
> fy := diff(f,y);
```

$$fy := 3\,y^2 - 3$$

Let Maple find the critical points.

```
> solve({fx=0,fy=0},{x,y});
```

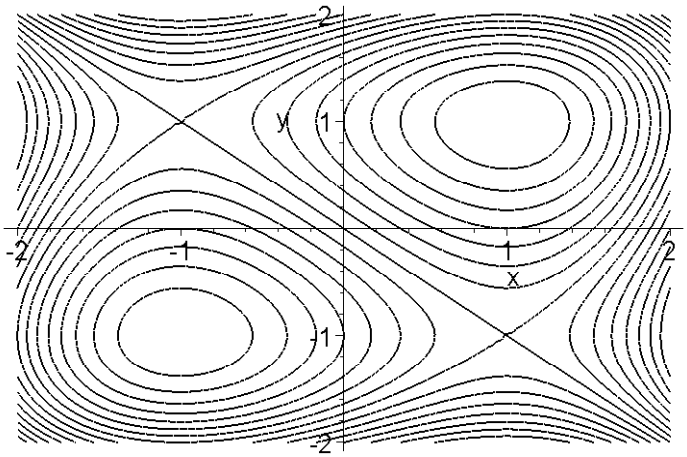
$$\{x = 1, y = 1\}, \{x = 1, y = -1\}, \{x = -1, y = 1\}, \{x = -1, y = -1\}$$

Create a contour diagram.

```
> c := [seq(i/2,i=-10..10)];
```

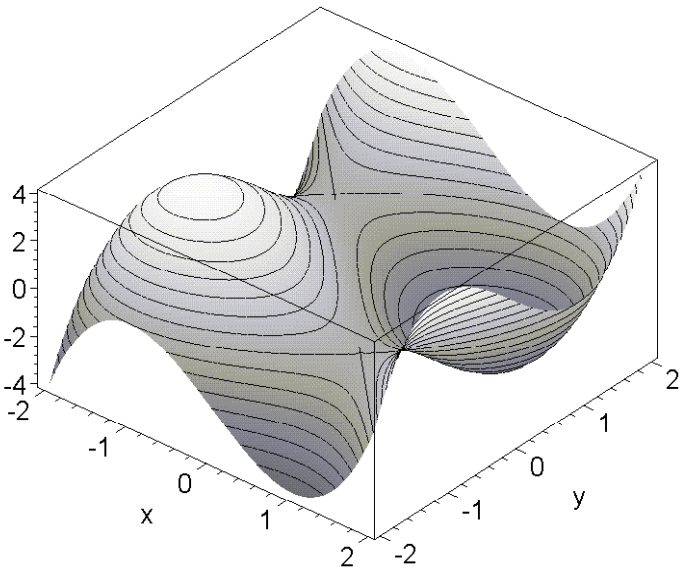
$$c := \left[-5, \frac{-9}{2}, -4, \frac{-7}{2}, -3, \frac{-5}{2}, -2, \frac{-3}{2}, -1, \frac{-1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}, 4, \frac{9}{2}, 5\right]$$

```
> contourplot(f,x=-2..2,y=-2..2,contours=c,grid=[80,80],color=black,
  thickness=2);
```



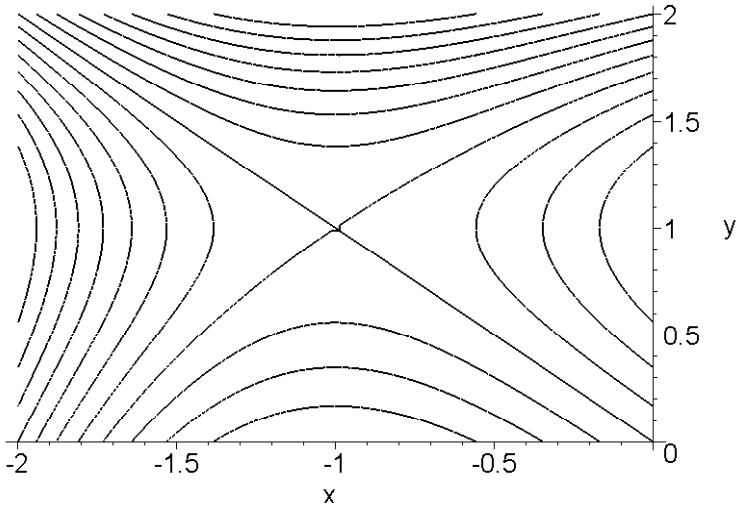

```
> plot3d(f,x=-2..2,y=-2..2,axes=boxed,style=patchcontour,grid=[60,60
```

```
],contours=c,orientation=[-50,40],shading=zgrayscale);
```

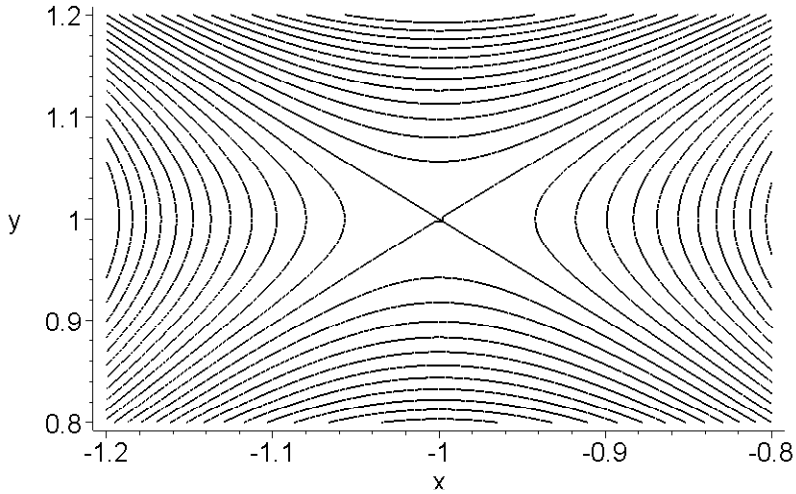


Take a closer look around x=-1, y=1.

```
> contourplot(f,x=-2..0,y=0..2,grid=[80,80],contours=c,color=black,thickness=2);
```

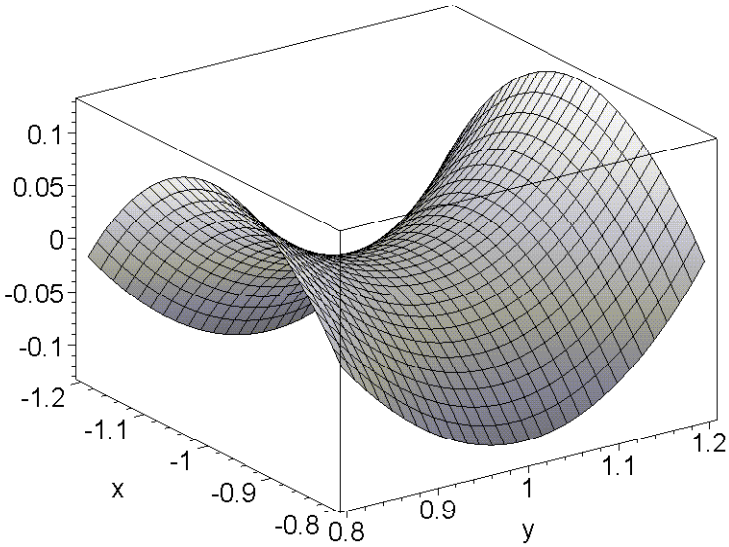



```
> contourplot(f,x=-1.2..-0.8,y=0.8..1.2,grid=[80,80],contours=25,color=black,thickness=2);
```



Note that as we zoom in on the critical point at (-1,1), the graph looks like a saddle.

```
> plot3d(f,x=-1.2..-0.8,y=0.8..1.2,grid=[30,30],shading=zgrayscale,axes=boxed,orientation=[-35,60]);
```




```
>
```