## Plotting Surfaces

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Plotting the graph of a function of two variables.
Let's plot the graph of $f(x, y)=x^{\wedge} 2-y^{\wedge} 2$. To do this, we'll use the plot3d command.

In its most basic form, the plot3d command takes three arguments. The first is the expression that defines the function to be plotted, and the next two are the ranges of the independent variables.
$>\operatorname{plot} 3 d\left(x^{\wedge} 2-y^{\wedge} 2, x=-1 \ldots 1, y=-1 \ldots 1\right)$;


Note: To see a different view, use the mouse to click and drag in the plot. To change some of the plot options, right-click on the plot, and select an option. For example, try using the mouse to add axes to the plot.

Just like the plot command, the plot3d command can be given many optional arguments. Look for plot3d in the Maple Help, or enter the command ?plot3d to learn more.

Here is the same command as above, with some options added directly to the command.
$>\operatorname{plot} 3 d\left(x^{\wedge} 2-y^{\wedge} 2, x=-1 \ldots 1, y=-1 \ldots 1\right.$, axes=boxed, grid=[64,64],style=patch contour) ;


With the "style=patchcontour" option, the lines drawn in the surface are contours lines instead of the xy grid. If you look at the plot "from above", you will see the contour diagram.

Here is another surface. Experiment with some of the plot options in the following command. For example, what does the contour diagram look like?
plot $3 d\left(x^{\wedge} 2+4 * y^{\wedge} 2, x=-1 \ldots 1, y=-1 \ldots 1\right.$, axes=boxed);


Exercise: Before going on, try plotting some different functions. For example, here is a "monkey saddle": $f(x, y)=x *(y-s q r t(3) * x) *(y+s q r t(3) * x)$.

## Surfaces Defined Implicitly

A surface can be defined implicitly, rather than as the graph of a function of two variables. That is, instead of defining the surface as $\mathrm{z}=\mathrm{f}(\mathrm{x}, \mathrm{y})$, it may be defined as a level set $\mathrm{g}(\mathrm{x}, \mathrm{y}, \mathrm{z})=\mathrm{C}$. There is a command in Maple, called implicitplot3d that can be used to plot such surfaces. To use it, we must first load a library of functions with the with (plots) : command. (Don't worry about the warning that appears after this command is entered.)
> with (plots):
Warning, the name changecoords has been redefined
Let's first plot something simple: the sphere $x^{\wedge} 2+y^{\wedge} 2+z^{\wedge} 2=1$.
> implicitplot3d ( $x^{\wedge} 2+y^{\wedge} 2+z^{\wedge} 2=1, x=-2.2, y=-2 \ldots 2, z=-2 \ldots 2$, grid= $[20,20,2$ 0], style=patchcontour, scaling=constrained, axes=boxed);


The set of points that are the same distance from the yz plane as they are from the x axis is given by the following equation:

$$
x^{\wedge} 2=y^{\wedge} 2+z^{\wedge} 2 .
$$

Let's plot this surface:
> implicitplot3d( $x^{\wedge} 2=y^{\wedge} 2+z^{\wedge} 2, x=-3 . .3, y=-3.3, z=-3.3$, grid=[25,25,25]
,style=patchcontour,scaling=constrained);


Let's plot the surface defined by
$x+x^{\wedge} 3-z^{\wedge} 2+2^{*} x^{*} y=-1$.
Here is the command:
$>$ implicitplot3d(x+y^3-z^2+2*x*y=-1,x=-3..3,y=-3..3,z=-3..3,grid=[25 ,25,25], style=patchcontour) ;


Try changing some of the coefficients in the equation to see how the surface changes.

