Final Exam   Math 112   May 3, 2005

NAME_____ 9:20, 10:20, 12:20
                                        Circle your class time.

- No calculator is allowed. Do not round off answers. If you get 10/6 as an answer, you should leave your answer as 5/3. No decimal approximations, please.

- If you have any questions, please raise your hand and ask. The worst that will happen is that I will say, "I can't tell you."

- Do the problems that you find easiest first. Take deep breaths between questions.

- There are 100 points on this exam, and you have 120 minutes.

- I hope you all do well. Good luck!

Question	Points	Points Earned
1	8	
2	8	
3	10	
4a	6	
4b	6	
4c	6	
4d	6	
4e	6	
5	10	
6	10	
7	12	
8	6	
9	6	
TOTAL	100	

Take three deep breaths.

What information have you been given?

What information do you need?

How can you get from one to the other?

Error Formulas:

$$|E_T| \leq \frac{K(b-a)^3}{12n^2}$$

$$|E_M| \leq \frac{K(b-a)^3}{24n^2}$$

$$|E_S| \leq \frac{K(b-a)^5}{180n^4}$$

1. (8 points) Identify whether each series converges or diverges. Justify your answer and show your work.

   (a) $\displaystyle\sum_{n=5}^{\infty} \frac{1}{n \ln(n)}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{n+2}{n+3}$

2. (8 points) Find the interval of convergence for the following series paying special attention to the endpoints of the interval.

   $$\sum_{n=1}^{\infty}(-1)^n \frac{x^n}{n 7^n}$$

3. (10 points) Evaluate the following limits. Justify your answer.

(a) $\lim\limits_{x \to 1} \dfrac{\ln(x)}{\cos(\pi x)}$

(b) $\lim\limits_{x \to 1} \dfrac{\ln(x)}{\sin(\pi x)}$

(c) $\lim\limits_{x \to 1} \dfrac{x^a - 1}{x^b - 1}$ for a and b arbitrary constant values with $b \neq 0$.

4. (30 points) Evaluate the following integrals:

(a) $\int \dfrac{dx}{x^2 + 4x + 13}$

(b) $\int_1^2 x^2 \ln(x) dx$

(c) $\int \sec^2(x) \tan^2(x) dx$

(d) $\int \dfrac{x^3 - 12x + 14}{x^2 + 3x - 4}\,dx$

(e) $\int \dfrac{x^2}{\sqrt{8x^2 + 2}}\,dx$

5. (10 points) A spherical container of radius 2m is full of water. Set up but DO NOT EVAL-UATE an integral for the work done pumping all the water to the top of the container. The density of water is $1000 kg/m^3$. If you use a picture, labels any axes you use. If you don't use a picture describe any symbol you introduce (like x or y). Your reader should be able to tell what every symbol refers to and what it means. Stop when your integral has one variable and is ready to integrate.

6. (10 points) Find the volume of the solid obtained by rotating the graph of $f(x) = e^{-2x}$ for $0 \leq x \leq \infty$ around the x-axis.

7. (12 points) Consider the curve given by $x = e^t \cos(t)$, $y = e^t \sin(t)$ for all real values of t.

a) Find the equation for the tangent line of the curve at the point (x, y) given by $t = 0$.

b) Find the length of this curve from $t = 0$ to $t = \pi/2$.

8. (6 points) Write the first three terms of the Taylor series for $f(x) = \frac{1}{x}$ about $a = 2$. (You don't have to find the expression for the general term or write it in summation form. Just write the first three terms.)

9. (6 points) Determine the sum of the given power series evaluated at $x = 1/3$. Explain how you obtain your answer. Hint: Find a function $f(x)$ which equals this series and the sum is then $f(1/3)$.

$$\sum_{n=0}^{\infty} nx^{n-1}$$