Brief Solutions to the Spring 2005 Math 112 Final Exam

1. (a) Diverges by the integral test, with $f(x) = \dfrac{1}{x\ln(x)}$.

   (b) Diverges by the test for divergence: $\lim_{n\to\infty} a_n \neq 0$.

2. The ratio test gives $R = 7$. At $x = -7$, we have the harmonic series, which is divergent. At $x = 7$, we have the alternating harmonic series, which is convergent. Thus the interval of convergence is $(-7, 7]$ (i.e. $-7 < x \leq 7$).

3. (a) 0

   (b) $-\dfrac{1}{\pi}$

   (c) $\dfrac{a}{b}$

4. (a) $\dfrac{1}{3}\arctan\left(\dfrac{x+2}{3}\right) + C$

   (b) $\dfrac{8\ln(2)}{3} - \dfrac{7}{9}$

   (c) $\dfrac{\tan^3(x)}{3} + C$

   (d) $\dfrac{x^2}{2} - 3x + \dfrac{3}{5}\ln|x-1| + \dfrac{2}{5}\ln|x+4| + C$

   (e) $\dfrac{1}{8\sqrt{2}}\left[x\sqrt{1+4x^2} - \dfrac{1}{2}\ln\left|2x + \sqrt{1+4x^2}\right|\right] + C$ **or**

   $\dfrac{x}{16}\sqrt{8x^2+2} - \dfrac{\sqrt{2}}{32}\ln\left|\dfrac{1}{2\sqrt{2}}\sqrt{8x^2+2} + x\right| + C$ **or...** (There are numerous equivalent forms.)

5. $W = \displaystyle\int_{-2}^{2} \pi 9800(4-y^2)(2-y)\,dy$

6. $\dfrac{\pi}{4}$

7. (a) $y = x - 1$

   (b) $\sqrt{2}\left(e^{\frac{\pi}{2}} - 1\right)$

8. $\dfrac{1}{2} - \dfrac{1}{4}(x-2) + \dfrac{1}{8}(x-2)^2 + \ldots$

9. $\dfrac{9}{4}$