Brief Solutions to the Spring 2005 Math 112 Final Exam

1. (a) Diverges by the integral test, with \( f(x) = \frac{1}{x \ln(x)} \).
   (b) Diverges by the test for divergence: \( \lim_{n \to \infty} a_n \neq 0 \).

2. The ratio test gives \( R = 7 \). At \( x = -7 \), we have the harmonic series, which is divergent. At \( x = 7 \), we have the alternating harmonic series, which is convergent. Thus the interval of convergence is \((-7, 7]\) (i.e. \(-7 < x \leq 7\)).

3. (a) 0
   (b) \(-\frac{1}{\pi}\)
   (c) \( \frac{a}{b} \)

4. (a) \( \frac{1}{3} \arctan \left( \frac{x + 2}{3} \right) + C \)
   (b) \( \frac{8 \ln(2)}{3} - \frac{7}{9} \)
   (c) \( \frac{\tan^3(x)}{3} + C \)
   (d) \( \frac{x^2}{2} - 3x + \frac{3}{5} \ln|x - 1| + \frac{2}{5} \ln|x + 4| + C \)
   (e) \( \frac{1}{8\sqrt{2}} \left[ x\sqrt{1 + 4x^2} - \frac{1}{2} \ln \left| 2x + \sqrt{1 + 4x^2} \right| \right] + C \) or
   \( \frac{x}{16} \sqrt{8x^2 + 2} - \frac{\sqrt{2}}{32} \ln \left| \frac{1}{2\sqrt{2}} \sqrt{8x^2 + 2} + x \right| + C \) or... (There are numerous equivalent forms.)

5. \( W = \int_{-2}^{2} \pi 9800(4-y^2)(2-y) \, dy \)

6. \( \pi \frac{4}{4} \)

7. (a) \( y = x - 1 \)
   (b) \( \sqrt{2 \left( e^x - 1 \right)} \)

8. \( \frac{1}{2} - \frac{1}{4}(x - 2) + \frac{1}{8}(x - 2)^2 + \ldots \)

9. \( \frac{9}{4} \)