Exam 3 Math 112 November 14, 2006

NAME_____

- No calculator is allowed.
- If you have any questions, please raise your hand and ask. The worst that will happen is that I will say, "I can't tell you."
- Do the problems that you find easiest first. Take deep breaths between questions.
- There are 100 points on this exam, and you have 110 minutes.
- I hope you all do well. Good luck!

Question	Points	Points Earned
1	13	
2	10	
3	10	
4	10	
5	10	
6	7	
7	10	
8	10	
9	10	
10	10	
TOTAL	100	

Take three deep breaths.

What information have you been given?

What information do you need?

How can you get from one to the other?

Trigonometric Product Formulas:

$\sin(A)\sin(B) = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

$\cos(A)\cos(B) = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

$\sin(A)\cos(B) = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

1. (14 points) Evaluate the following limits. Show your work.

a) $\lim_{x \to 0^+} \dfrac{1}{e^x - 1} - \dfrac{1}{x}$

b) $\lim_{x \to \infty} \left(\dfrac{x-2}{x} \right)^x$

c) $\lim_{\theta \to 0} \dfrac{\sin(\theta)}{1 + \cos(\theta)}$

2. (10 points) Evaluate the integral if it converges, otherwise show it diverges.
$$\int_2^\infty \frac{dx}{(x-7)^4}$$

3. (10 points) Evaluate the integral if it converges, otherwise show it diverges.
$$\int_4^\infty \frac{dx}{x\sqrt{x-1}}$$

4. (10 points) Use the Comparison Theorem to determine whether or not the following integral is convergent. (You do not need to evaluate the integral.)
$$\int_1^\infty \frac{1}{3x^2 + 2\sin(2x) + 2} dx$$

5. (6 points) Find the (x, y) coordinates of all the points of intersection for the curves $r = \sin(\theta)$ and $r = \cos(\theta)$.

6. (10 points) Set up BUT DO NOT EVALUATE an integral for the area of the region that is inside both $r = \sin(\theta)$ and $r = 1 - \sin(\theta)$.

7. (10 points) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ for the curve $x(t) = 3t^5 + 4t + 5$, $y(t) = 3t^2 + 2t + 3$ at the point $(x, y) = (5, 3)$.

8. (10 points) Consider the cochleoid given by $r = \begin{cases} \frac{\sin(\theta)}{\theta} & \theta \neq 0 \\ A & \theta = 0 \end{cases}$.

a) Find the value for A which makes this function a continuous function.

b) Sketch the resulting curve for $0 \leq \theta \leq 2\pi$.

9. (10 points) For the following polar curves find the equation for the curve in cartesian coordinates. Then identify the curve (i.e. using English).

   a) $\tan(\theta) = 5$

   b) $r = \dfrac{1}{1 + \cos(\theta)}$.

10. (10 points) Find the length of the curve given in polar coodinates as $r = e^{\theta/2}$, for $0 \leq \theta \leq 2\pi$.