Answers

1) $\lim\limits_{x\to 0^-} \dfrac{\cos(x)}{x} = \boxed{\lim_{x\to 0^-} \frac{1}{x} = -\infty}$

2) $\lim\limits_{x\to 0^+} x^{\arctan(x)} = \boxed{\lim_{x\to 0^+} e^{\ln\left(x^{\arctan(x)}\right)} = \ldots = 1}$ (Several applications of L'Hospitals rule.)

3) $\lim\limits_{x\to 2^+} \dfrac{\ln(x-1)}{x^2 + 4x - 12} = \boxed{\text{L'Hospitals rule} = 1/8}$

4) $\int_{-\infty}^{0} e^{-2x} dx \;\boxed{\text{Diverges}}$.

5) $\int_{0}^{2} \dfrac{x^3}{x^2 - 1} dx \;\boxed{\text{Diverges}}$.

6) Part one: Short answer. For what values of p does the integral $\int_{1}^{\infty} \dfrac{1}{x^p} dx$ diverge? $\boxed{\text{For } p \le 1}$.

Part two: Use the comparison theorem to determine whether the following integral converges.

$\int_{3}^{\infty} \dfrac{\cos^2 x}{x^3} dx \;\boxed{\text{Converges}}$.

7) Your company is trying to predict its total revenue for next year. The total revenue is equal to $\int_{0}^{66} f(t)\, dt$. The values in the table below give $f(t)$ for various values of t. Approximate the true value of $\int_{0}^{66} f(t)\, dt$ using first the Trapazoidal Rule and then Simpson's Rule.

$\boxed{\Delta x = 11, \text{ so } T_6 = 11\,(2.1/2 + 4.2 + 3.6 + 8.2 + 6.1 + 0.2 + 1.2/2).}$

$\boxed{S_6 = \frac{11}{3}\,(2.1 + 4(4.2) + 2(3.6) + 4(8.2) + 2(6.1) + 4(0.2) + 1.2).}$

8) Estimate the integral $\int_{0}^{1} -\ln(1+t)dt$ using the trapezoidal rule to within 0.001.

a) Find a value of K that works in this problem. Explain with equations and/or words. $\boxed{K=1}$

b) Suppose after doing the calculations, we obtain the inequality: $|E_T| \le \dfrac{0.1}{n^2}$. Using this inequality, how many intervals do you need to ensure the desired accuracy. $\boxed{n=10}$.

9) Find the equations for both of the tangent lines at the crossing $(x,y) = (1,2)$ of the curve $x = t^3 - t + 1$, $y = 3 - t^2$, $-\infty \le t \le \infty$.

$\boxed{\text{The } t \text{ values are } t = \pm 1 \text{ for } (x,y) = (1,2).\; \dfrac{dy}{dx} = \dfrac{-2t}{3t^2 - 1}.\; \text{The equations are: } y - 2 = \pm(x-1).}$

10) Find the area under the curve for $0 \le x \le \ln(3)$ where the curve is given by $x(t) = \ln(t+2)$, and $y(t) = (t+1)/(3-t)$.

$\boxed{A = \int_{0}^{\ln 3} y\, dx = \int_{-1}^{1} \dfrac{t+1}{(3-t)(t+2)} dt = -\dfrac{4}{5}(\ln 2 - \ln 4) - \dfrac{1}{5}(\ln 3 - \ln 1) = \dfrac{4}{5}\ln 2 - \dfrac{1}{5}\ln 3}$.

11) Find the area inside the cardioid $r = 1 + \cos(\theta)$ $\boxed{= \frac{3\pi}{2}}$.

12) Set up an integral for the length of the polar curve $r = \sin^3\left(\frac{\theta}{3}\right)$ for $0 \le \theta \le \frac{\pi}{2}$.

$\boxed{\int_{0}^{\pi/2} \sqrt{\sin^4(\theta/3)\cos^2(\theta/3) + \sin^6(\theta/3)}\, d\theta}$