Test 3 Math 112 April 12, 2005

NAME_____ 9:20, 10:20, 12:20
Circle your class time.

- No calculator is allowed. Do not round off answers. If you get 10/6 as an answer, you should leave your answer as 5/3. No decimal approximations, please.
- For problems with numerical approximation of integrals, write your answer as an expression involving only numbers (no symbols). You do not need to evaluate that expression.
- If you have any questions, please raise your hand and ask. The worst that will happen is that I will say, "I can't tell you."
- Do the problems that you find easiest first. Take deep breaths between questions.
- There are 100 points on this exam, and you have 110 minutes.
- I hope you all do well. Good luck!

| Question | Points | Points Earned |
|---|---|---|
| 1 | 6 | |
| 2 | 7 | |
| 3 | 6 | |
| 4 | 9 | |
| 5 | 9 | |
| 6 | 9 | |
| 7 | 9 | |
| 8 | 9 | |
| 9 | 9 | |
| 10 | 9 | |
| 11 | 9 | |
| 12 | 9 | |
| TOTAL | 100 | |

Take three deep breaths.

What information have you been given?

What information do you need?

How can you get from one to the other?

Error Formulas:

$$|E_T| \leq \frac{K(b-a)^3}{12n^2}$$

$$|E_M| \leq \frac{K(b-a)^3}{24n^2}$$

$$|E_S| \leq \frac{K(b-a)^5}{180n^4}$$

1. (6 points) Show your work. Evaluate $\lim_{x \to 0^-} \dfrac{\cos(x)}{x}$.

2. (7 points) Show your work. Evaluate $\lim_{x \to 0^+} x^{\arctan(x)}$.

3. (6 points) Show your work. Evaluate $\lim_{x \to 2^+} \dfrac{\ln(x-1)}{x^2+4x-12}$.

4. (9 points) Show your work. Evaluate $\int_{-\infty}^{0} e^{-2x} dx$.

5. (9 points) Show your work. Evaluate $\int_{0}^{2} \frac{x^3}{x^2 - 1} dx$.

6. (9 points) Part one: Short answer. For what values of $p$ does the integral $\int_{1}^{\infty} \frac{1}{x^p}\, dx$ diverge?

   Part two: Use the comparison theorem to determine whether the following integral converges. Explain your thoughts with equations and/or words.
   $\int_{3}^{\infty} \frac{\cos^2 x}{x^3}\, dx$

7. (9 points) Your company is trying to predict its total revenue for next year. The total revenue is equal to $\int_0^{66} f(t)\,dt$. (By the way, the function $f(t)$ is called the income stream.) The values in the table below give $f(t)$ for various values of $t$. Approximate the true value of $\int_0^{66} f(t)\,dt$ using first the Trapazoidal Rule and then Simpson's Rule. Label your answers clearly, and stop as soon as there are no letters (only numbers) in your answer.

| t | 0 | 11 | 22 | 33 | 44 | 55 | 66 |
|---|---|---|---|---|---|---|---|
| f(t) | 2.1 | 4.2 | 3.6 | 8.2 | 6.1 | 0.2 | 1.2 |

8. (9 points) The goal in this problem is to find how many intervals are needed to estimate the integral $\int_0^1 -\ln(1+t)dt$ using the trapezoidal rule to within 0.001.

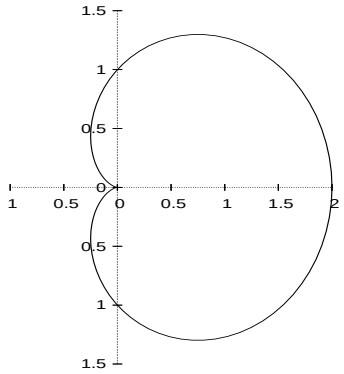
   a) Find a value of $K$ that works in this problem. Explain with equations and/or words.

   b) Suppose after doing the calculations, we obtain the inequality: $|E_T| \leq \frac{0.1}{n^2}$. Using this inequality, how many intervals do you need to ensure the desired accuracy.

9. (9 points) Show your work. Find the equations for both of the tangent lines at the crossing $(x, y) = (1, 2)$ of the curve $x = t^3 - t + 1$, $y = 3 - t^2$, $-\infty \leq t \leq \infty$.

10. (9 points) Show your work. Find the area under the curve for $0 \leq x \leq \ln(3)$ where the curve is given by $x(t) = \ln(t + 2)$, and $y(t) = (t + 1)/(3 - t)$.

11. (9 points) Show your work. Find the area inside the carteoid $r = 1 + \cos(\theta)$.




12. (9 points) Set up but DO NOT EVALUATE an integral for the length of the polar curve $r = \sin^3(\frac{\theta}{3})$ for $0 \leq \theta \leq \frac{\pi}{2}$. Stop when you have removed all $r$'s from the expression.