Plotting Surfaces with *Grapher*

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These brief notes explain how to plot surfaces with *Grapher*, an application that is available on the computers in the Computer Lab.

Plotting the graph of $z = f(x, y)$.

1. Start *Grapher* by clicking on the *Grapher* icon. A window with the title **New Graph** will appear.

2. Click on **3D Graph**, then click on **Open**. A larger window with the title **Untitled** will appear. There will be an empty set of yellow axes visible. Above the axes is an area where you can define your equation. It will initially contain $z =$. You can type your function here.

3. After $z =$, enter, for example, $(x - 2y)(x + 2y)x$. When you hit Enter, the graph will be drawn with a light and dark green checkerboard pattern.

   Note: When you type the first opening parenthesis, the program automatically adds the closing parenthesis, and puts the cursor inside them. You can use the right-arrow key to move out of the parentheses.

4. The surface will looked "stretched out" vertically. A quick way to see a nicer view is to click on **Zoom In** two or three times.

   Note: This plot is an example of a "monkey saddle".

5. Move the mouse into the plotting region, and click-and-drag on the plot to change the point of view.

6. *Optional, but possibly useful:* We can control the view more precisely by changing the "Frame Limits". In the menu bar at the top of the screen, select **View**, and then **Frame Limits...**. In the small window that pops up, you can set the x, y and z limits of the plot. Change the x and y lower and upper limits to be, say, -1 and 1, respectively, and change the z limits to be -2 and 2. Click on **OK** (or just hit Enter).

7. *Grapher* can show the contour lines of this function. In the menu bar at the top of the screen, select **Window**, and then **Show Inspector**. The small window that appears allows you to change several aspects of the plot. In particular, click on the check box next to **Contours**. This will add contour lines in the surface. These are the curves where horizontal planes intersect the surface. If you then rotate the surface so that you are looking straight down the z axis, you will see the contour diagram.

   Click on the small left-most circle in the title bar of the **Inspector** window to close it.

8. Experiment with other functions. For example, try the function from the quiz: $z = xy^2$. While you are at it, experiment with other options in the **Inspector** window. Explore other features of the program. Have fun!

Plotting implicitly defined surfaces. *Grapher* can handle surfaces defined implicitly. For example, the equation of a sphere of radius 2 centered at the origin is

$$x^2 + y^2 + z^2 = 4$$

1. Enter the equation of the sphere in the field above the plot. (You can first delete anything that is already there, including $z =$.)

   Note: Depending on your current Frame Limits, you may first have to click on **Zoom Out** a few times to see the sphere.

2. The sphere that you see is not perfectly smooth. It is faceted, and looks a bit like a golf ball. The plot generated by the computer is only an approximation, in which the smooth surface is approximated with many small flat surfaces. You can control resolution of the approximation. Select **Window**, and **Show Inspector** (if the **Inspector** window is not already visible), and in the section labeled **Implicit Graph**, move the blue sliders next to **Abscissa**, **Ordinate**, and **Height** to the right. Then click on the black circular arrow to regenerate the plot. It may take a few seconds, but eventually the surface will be redrawn, and the facets will be smaller.

3. Experiment with other implicitly defined surfaces. For example, try this one:

$$x + x^3 - z^2 + 2xy = -1.$$

   (Use x, y and z frame limits of -3 and 3.) Then change $-z^2$ to $+z^2$ and observe how the surface changes. Make up your own formulas. Go nuts.