Math 113 Fall '05

Homework 3 Solutions

Note: some of the problems are out of order.

$\boxed{12.4/2}$

(a) Since z is a linear function of x and y with slope 2 in the x direction and slope 3 in the y direction, we have

$$z = f(x,y) = 2x + 3y + c$$

Now, in general

$$f(x+\Delta x, y+\Delta y) = 2x + 2\Delta x + 3y + 3\Delta y + c$$

so

$$\Delta z = f(x+\Delta x, y+\Delta y) - f(x,y) = 2\Delta x + 3\Delta y$$

If $\Delta x = 0.5$ and $\Delta y = -0.2$, we have

$$\Delta z = 2(0.5) + 3(-0.2) = 0.4$$

That is, the change in z is **0.4.**

(b) We have $\Delta x = 4.9 - 5 = -0.1$, and $\Delta y = 7.2 - 7 = 0.2$

so $\Delta z = 2(-0.1) + 3(0.2) = 0.4$.

Thus the value of z when $x = 4.9$ and $y = 7.2$ is

$$z = 2 + \Delta z = \mathbf{2.4}$$

(value of z when $x=5$ and $y=7$)

[redacted]

$\boxed{12.4/8}$

The points $(0,0,0)$ and $(-3,0,-4)$ are in the xz-plane, so we can use them to find m:

$$m = \frac{\Delta z}{\Delta x} = \frac{-4-0}{-3-0} = \frac{4}{3}$$

The points $(0,0,0)$ and $(0,2,-1)$ are in the yz-plane, so we can use them to find n:

$$n = \frac{\Delta z}{\Delta y} = \frac{-1-0}{2-0} = -\frac{1}{2}$$

When $x=0$ and $y=0$, we see that $z=0$, because $(0,0,0)$ is in the plane. Thus $c=0$.

The equation is therefore

$$z = \frac{4}{3}x - \frac{1}{2}y$$

$\boxed{12.4/9}$ (NOT ASSIGNED)

$(4,0,0)$ and $(0,0,2)$ are in the xz-plane, so

$$m = \frac{\Delta z}{\Delta x} = \frac{2-0}{0-4} = -\frac{1}{2}$$

$(0,0,2)$ and $(0,3,0)$ are in the yz-plane, so

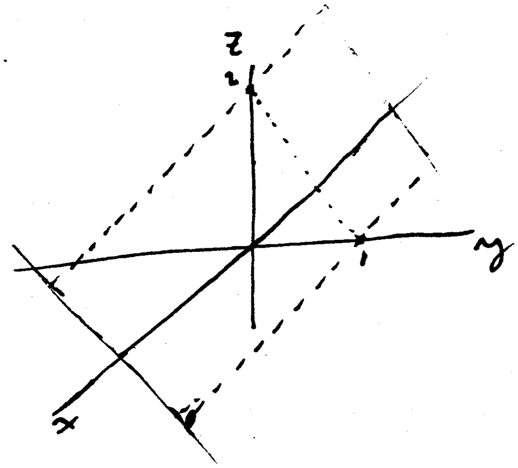
$$n = \frac{\Delta z}{\Delta y} = \frac{0-2}{3-0} = -\frac{2}{3}$$

Because $(0,0,2)$ is in the plane, we must have $c=2$.

Thus the equation is

$$z = -\frac{1}{2}x - \frac{2}{3}y + 2$$

$\boxed{12.4/10}$



Because the plane contains lines that are parallel to the x axis, the slope in the x direction must be zero: $m = 0$.

The points $(0,0,2)$ and $(0,1,0)$ are in the plane, and in the yz-plane, so

$$n = \frac{\Delta z}{\Delta y} = \frac{0-2}{1-0} = -2$$

Because $(0,0,2)$ is in the plane, we see that $c = 2$.

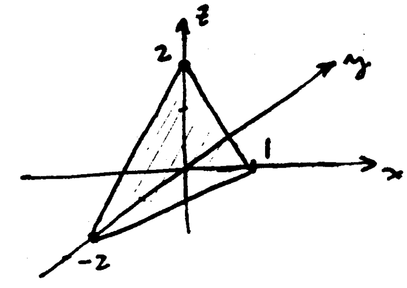
Thus the equation is

$$z = -2y + 2$$

$\boxed{12.4/21}$ - Next page →

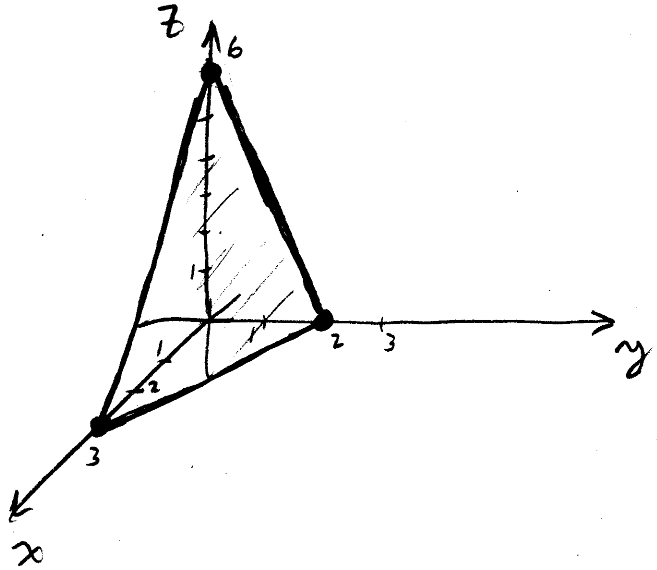
$\boxed{12.4/23}$

If $x=0$ and $y=0$ then $z=2$, so $(0,0,2)$ is the z-intercept
If $x=0$ and $z=0$, then $y=-2$, so $(0,-2,0)$ is the y-intercept
If $y=0$ and $z=0$, then $x=1$, so $(1,0,0)$ is the x-intercept.



$\boxed{12.4/21}$

The intercepts are $(3,0,0)$, $(0,2,0)$, and $(0,0,6)$.



$\boxed{12.4/12}$ The contour values (ie the z values) are evenly spaced (they increase by 2), but the contour lines are not evenly spaced, so this contour diagram could not represent a linear function.

$\boxed{12.4/13}$ The contour lines are evenly spaced parallel lines, so this diagram could represent a linear function.

$\boxed{12.4/14}$ Yes (see answer to #13).

$\boxed{12.4/15}$ No (see answer to #12).

(NOT ASSIGNED)

$\boxed{12.4/18}$ A linear function has the form $f(x,y) = mx + ny + c$.

The table shows that $f(0,0) = 1$, so $\boxed{c = 1}$.

To find m, the slope in the x direction, we compute $m = \frac{\Delta z}{\Delta x}$ using any two points that have the same y value. For example, we have $f(0,0) = 1$ and $f(1,0) = 3$, so two points in the graph are $(0,0,1)$ and $(1,0,3)$. Then $\boxed{m = \frac{\Delta z}{\Delta x} = \frac{3-1}{1-0} = 2}$

Similarly, $n = \frac{\Delta z}{\Delta y}$ (with constant x), and we have $f(0,0) = 1$, $f(0,2) = 0$. Then $\boxed{n = \frac{\Delta z}{\Delta y} = \frac{0-1}{2-0} = -\frac{1}{2}}$

so the linear function is $\boxed{f(x,y) = 2x - \frac{1}{2}y + 1.}$

$\boxed{12.5/2}$ The level surfaces are cylinders around the z axis. A possible function is

$$f(x,y,z) = x^2 + y^2$$

NOTE — OUT OF ORDER!

$\boxed{12.5/8}$ $z - x^2 - 3y^2 = 0$

Solve for z:

$$z = x^2 + 3y^2$$

Since we can solve for z, the surface can be expressed as the graph of $f(x,y) = x^2 + 3y^2$.

$\boxed{12.5/6}$ $2x + 3y - 5z - 10 = 0$

Solve for z:

$$z = \frac{2}{5}x + \frac{3}{5}y - 2$$

We can solve for z, so the graph can be expressed as the graph of $f(x,y) = \frac{2}{5}x + \frac{3}{5}y - 2$

$\boxed{12.5/7}$ $x^2 + y^2 + z^2 - 1 = 0$

If we try to solve for z, we get $z = \pm\sqrt{1 - x^2 - y^2}$.
There are two values of z for every (x,y) on the surface.
We can not express this surface as the graph of a function $f(x,y)$.

$\boxed{12.5/9}$ Solving for z, we find $z = \pm\sqrt{x^2 + 3y^2}$. Each (x,y) gives two z values, so we cannot express this surface as the graph of a function $f(x,y)$.

$\boxed{12.5/10}$

The equation of a sphere of radius 2 centered at the origin is $x^2+y^2+z^2=2^2=4$

so we can simply choose $\underline{f(x,y,z)=x^2+y^2+z^2}$,

so $f(x,y,z)=4$ gives the sphere.

$\boxed{12.5/12}$

Just solve for z:

$$x+2y+3z=5 \Rightarrow z=\frac{1}{3}(5-x-2y)$$
$$=\frac{5}{3}-\frac{1}{3}x-\frac{2}{3}y$$

so $\underline{f(x,y)=\frac{5}{3}-\frac{1}{3}x-\frac{2}{3}y}$

$\boxed{12.5/14}$

The following are graphs of a function of x and y

$\underline{12.70,\ 12.71,\ 12.76,}$

$\boxed{12.5/15}$ $-x^2-y^2+z^2=1 \Rightarrow x^2+y^2-z^2=-1$

This matches Figure 12.74, with $a=b=c=1$.

The surface is a hyperboloid of two sheets.

$\boxed{12.5/16}$ see the next page.

$\boxed{12.5/17}$ $x^2+y^2-z=0 \Rightarrow z=x^2+y^2$

The surface is an elliptical paraboloid - Fig. 12.70.

$\boxed{12.5/18}$ $x^2+z^2=1$

The y variable is missing, so this is a cylindrical surface. The surface intersects the xz plane in a circle with radius 1. Thus the surface is a circular cylinder around the y axis. (This is like Fig. 12.77, but with y and z interchanged.)

$\boxed{12.5/16}$ $-x^2+y^2-z^2=0$

$\Rightarrow x^2-y^2+z^2=0$

We have a sum of squares, one of which has a different sign than the other two, and the constant term is zero. This has the form of Fig 12.75, except that the cone is around the y axis.

$\boxed{12.5/24}$

Two planes are parallel if they have the same slope in the x direction and the same slope in the y direction. Thus any plane of the form

$$z = 2x + 3y + c$$

is parallel to the given plane. We can rewrite this as

$$z - 2x - 3y = c,$$

which shows that we can interpret these planes as the level surfaces of

$$\boxed{g(x,y,z) = z - 2x - 3y}$$

$\boxed{12.5/26}$

$$g(x,y,z) = e^{-(x^2+y^2+z^2)}$$

so $g(x,y,z) = c \Rightarrow e^{-(x^2+y^2+z^2)} = c$

$$\Rightarrow -(x^2+y^2+z^2) = \ln c$$

$$x^2+y^2+z^2 = -\ln c$$

so the level surfaces are <u>spheres</u>.

$\boxed{12.5/27}$

$$f(x,y) = \sqrt{1-x^2-y^2}$$

(a) The graph is given by $z = \sqrt{1-x^2-y^2}$

$$\Rightarrow z^2 = 1-x^2-y^2 \qquad (z \geq 0)$$

$$\Rightarrow x^2+y^2+z^2 = 1 \qquad (z \geq 0)$$

This gives a sphere of radius <u>1</u> centered at the origin, but since $z \geq 0$, we only have the upper hemisphere.

(b) Use the idea (and formula) given on p. 594:

$$g(x,y,z) = f(x,y) - z$$

so in this case $g(x,y,z) = \sqrt{1-x^2-y^2} - z$. Then the surface is given by $g(x,y,z) = 0$ (i.e. $c = 0$).

$\boxed{12.5/30}$

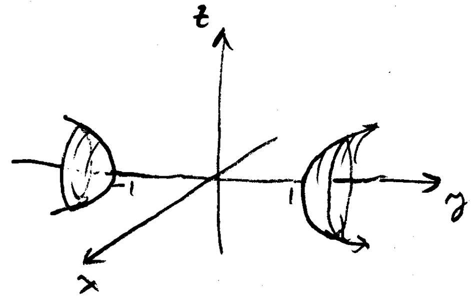
$f(x,y,z) = x^2 - y^2 + z^2$

The level surfaces are given by

$$x^2 - y^2 + z^2 = c$$

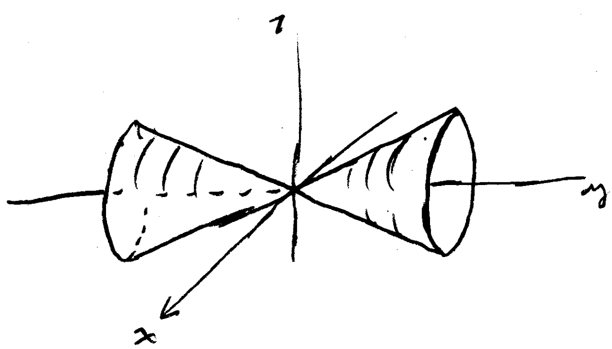
If $c<0$, this is a hyperboloid of two sheets, around the y axis:

(see Fig 12.74)



If $c=0$, this is a cone around the y axis

(see Fig. 12.75)



If $c>0$, this is a hyperboloid of one sheet around the y axis

(see Fig 12.73)