Math 113

Fall '05

Homework 4 Solutions

$\boxed{12.6/4}$

The functions $\sin x$ and e^x are continuous everywhere, and so is the composition $e^{\sin x}$. The function $\cos y$ is also continuous for all y. Therefore, the only points where the quotient $\frac{e^{\sin x}}{\cos y}$ is not continuous is where $\cos y = 0$. That is, where $y = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

or $y = -\frac{\pi}{2}, -\frac{3\pi}{2}, -\frac{5\pi}{2}, \ldots$

The region given in the problem is $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, $0 \leq y \leq \frac{\pi}{4}$, and $\cos y \neq 0$ on this region, so the function

$\frac{e^{\sin x}}{\cos y}$ <u>is</u> continuous at all points in the region.

$\boxed{12.6/12}$ Let $f(x,y) = \dfrac{2x - y^2}{2x + y^2}$.

Along the positive x axis, we have $y = 0$, and

$$z = f(x,0) = \frac{2x}{2x} = 1 \qquad (x \neq 0)$$

and so $\displaystyle\lim_{x \to 0} f(x,0) = 1.$

Along the positive y axis, we have $x = 0$, and

$$z = f(0,y) = \frac{-y^2}{y^2} = -1 \qquad (y \neq 0)$$

and so $\displaystyle\lim_{y \to 0} f(0,y) = -1$

Since the function approaches <u>different values</u> along different paths to $(0,0)$, the limit $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.

$\boxed{12.6/18}$ $f(x,y) = \frac{x^2 y}{x^4 + y^2}$ $(x,y) \neq (0,0)$

Along the line $y = x$, we have $f(x,x) = \frac{x^3}{x^4 + x^2}$

and $\lim_{x \to 0} f(x,x) = \lim_{x \to 0} \frac{x^3}{x^4 + x^2} = \lim_{x \to 0} \frac{\cancel{x^2}(x)}{\cancel{x^2}(x^2+1)}$

$$= \lim_{x \to 0} \frac{x}{x^2+1} = 0.$$

Along the curve $y = x^2$, we have

$$f(x, x^2) = \frac{x^4}{x^4 + x^4} = \frac{1}{2} \quad (\text{if } x \neq 0),$$

so $\lim_{x \to 0} f(x, x^2) = \lim_{x \to 0} \frac{1}{2} = \frac{1}{2}.$

We see that along these two paths, the function approaches different values, therefore

$\lim_{(x,y) \to (0,0)} f(x,y)$ does not exist.

Note that along any straight line $y = mx$, we have

$$f(x, mx) = \frac{mx^3}{x^4 + m^2 x^2} = \frac{mx}{x^2 + m^2} \quad (\text{if } x \neq 0)$$

Then $\lim_{x \to 0} f(x, mx) = \lim_{x \to 0} \frac{mx}{x^2 + m^2} = 0.$ So the function approaches 0 along any straight line through the origin, but along the parabola $y = x^2$, the function approaches $\frac{1}{2}$! (Now see what happens along other parabolas. Try $y = mx^2$.)