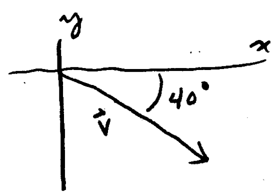
Math 113, Fall '05 — Homework 6 Solutions

$\boxed{13.1/12}$ $\vec{z} = \vec{i} - 3\vec{j} - \vec{k}$, $\|\vec{z}\| = \sqrt{1^2 + (-3)^2 + (-1)^2} = \sqrt{11}$

$\boxed{13.1/14}$
$$2\vec{a} + 7\vec{b} - 5\vec{z} = 2(2\vec{j} + \vec{k}) + 7(-3\vec{i} + 5\vec{j} + 4\vec{k}) - 5(\vec{i} - 3\vec{j} - \vec{k})$$
$$= (-21 - 5)\vec{i} + (4 + 35 + 15)\vec{j} + (2 + 28 + 5)\vec{k}$$
$$= -26\vec{i} + 54\vec{j} + 35\vec{k}$$

$\boxed{13.1/16}$



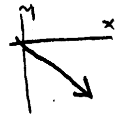
$$\vec{v} = 8\cos(-40°)\vec{i} + 8\sin(-40°)\vec{j}$$
$$= 8\cos(40°)\vec{i} - 8\sin(40°)\vec{j}$$
$$\approx 6.128\vec{i} - 5.142\vec{j}$$

$\boxed{13.1/20}$ $\vec{u} = \vec{i} + \vec{j} + 2\vec{k}$, $\vec{v} = -\vec{i} + 2\vec{k}$

$\boxed{13.1/22}$ A unit vector in the same direction is $\frac{1}{\sqrt{6}}(\vec{i} - \vec{j} + 2\vec{k})$, so a vector with length 2 in the same direction is
$$\frac{2}{\sqrt{6}}(\vec{i} - \vec{j} + 2\vec{k}).$$

$\boxed{13.1/24}$ $\vec{v} = 3\vec{i} + y\vec{j}$, $\|\vec{v}\| = \sqrt{9 + y^2} = 5 \Rightarrow y^2 = 16 \Rightarrow y = 4$ or $y = -4$
so there are two possible vectors, $\boxed{3\vec{i} + 4\vec{j}}$ and $\boxed{3\vec{i} - 4\vec{j}}$.

$\boxed{13.2/8}$



$10\cos(-\frac{\pi}{4})\vec{i} + 10\sin(-\frac{\pi}{4})\vec{j} = \boxed{5\sqrt{2}\,\vec{i} - 5\sqrt{2}\,\vec{j}}$

$\boxed{13.2/13}$

Let $\vec{v} = a\vec{i} + b\vec{j}$ be the velocity of the plane relative to the air, and let $\vec{w} = 60\cos(-\frac{\pi}{4})\vec{i} + 60\sin(-\frac{\pi}{4})\vec{j}$ $= 30\sqrt{2}\,\vec{i} - 30\sqrt{2}\,\vec{j}$ be the velocity of the air relative to the ground. Then the velocity of plane relative to the ground is $\vec{v} + \vec{w} = (a + 30\sqrt{2})\vec{i} + (b - 30\sqrt{2})\vec{j}$.

For the plane to end up heading due east, we must have $b - 30\sqrt{2} = 0 \Rightarrow b = 30\sqrt{2}$

so $\vec{v} = a\vec{i} + 30\sqrt{2}\,\vec{j}$.

We are told that the airspeed is 500, which means $\|\vec{v}\| = 500$.

Thus $\sqrt{a^2 + 1800} = 500 \Rightarrow a = \sqrt{(2500)^2 - 1800} = 10\sqrt{2482} \approx 498.2$

so $\vec{v} = 10\sqrt{2482}\,\vec{i} + 30\sqrt{2}\,\vec{j} \approx 498.2\vec{i} + 42.43\vec{j}$

The angle of this vector is

$\theta = \arctan\left(\frac{30\sqrt{2}}{10\sqrt{2482}}\right) \approx 0.08495$ radians ≈ 4.868 degrees (north of east)

The speed relative to the ground is

$\|\vec{v} + \vec{w}\| = a + 30\sqrt{2} = 10\sqrt{2482} + 30\sqrt{2} \approx 540.6$ km/hr

$\boxed{13.2/14}$ Let $\vec{v}$ be the velocity of the plane relative to the air. We know $\|\vec{v}\| = 480$ km/hr, and $\vec{v} = x\vec{i} + 80\vec{k}$ because the plane is heading due east. Then $\sqrt{x^2+80^2} = 480 \Rightarrow x = \sqrt{480^2 - 80^2} = \sqrt{224000}$
≈ 473.3

The velocity of the wind relative to the ground is

$$\vec{w} = 50\sqrt{2}\,\vec{i} + 50\sqrt{2}\,\vec{j} \approx 70.71\vec{i} + 70.71\vec{j}.$$

So the velocity of the plane relative to the ground is

$$\vec{v} + \vec{w} = (473.3 + 70.71)\vec{i} + 70.71\vec{j} + 80\vec{k}$$
$$= 544.0\vec{i} + 70.71\vec{j} + 80\vec{k}$$

The ground speed is the magnitude of this vector:

$$\sqrt{544.0^2 + 70.71^2 + 80^2} = \boxed{554.4 \text{ km/hr}}$$

(Note: If one interprets "ground speed" to mean just the horizontal component of the velocity, we obtain
$$\sqrt{544.0^2 + 70.71^2} = 548.6 \text{ km/hr}.)$$

13.2/23 The force exerted on the object from the first rope $\vec{F}_1 = 100\cos(30°)\vec{i} + 100\sin(30°)\vec{j} = 86.60\vec{i} + 50\vec{j}$ and the force exerted from the second rope is $\vec{F}_2 = 70\cos(80°)\vec{i} - 70\sin(80°)\vec{j} = 12.16\vec{i} - 68.94\vec{j}$. The sum of these two forces is $\vec{F}_1 + \vec{F}_2 = 98.76\vec{i} - 18.94\vec{j}$. See Figure 13.14. In order for the object to move vertically, the total force on the object must be in the form $\vec{F} = 0\vec{i} + 0\vec{j} + b\vec{k}$ for some b. Thus the force vector for the crane is

$$\vec{F}_c = -98.76\vec{i} + 18.94\vec{j} + b\vec{k}$$

for some b. To find b, we use the fact that $||\vec{F}_c|| = 3000$. Thus,

$$\begin{aligned} ||\vec{F}_c|| &= 3000 \\ \sqrt{(98.76)^2 + (18.94)^2 + b^2} &= 3000 \\ b &= \pm 2998.31 \end{aligned}$$

We use the positive value of b since we want the object to go up rather than down. The force exerted by the crane is

$$\vec{F}_c = -98.76\vec{i} + 18.94\vec{j} + 2998.31\vec{k}.$$

The total force acting on the object is $2998.31\vec{k}$, or 2998.31 newtons straight up.

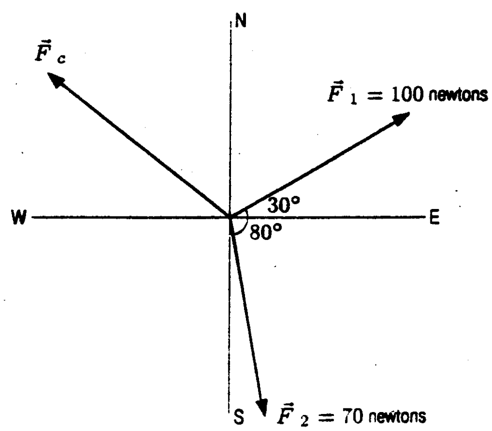


Figure 13.14: Horizontal forces on object