

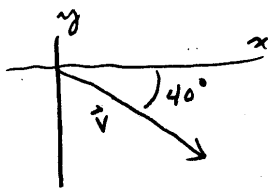
13.1/12

$$\vec{z} = \vec{i} - 3\vec{j} - \vec{k}, \quad \|\vec{z}\| = \sqrt{1^2 + (-3)^2 + (-1)^2} = \sqrt{11}$$

13.1/14

$$\begin{aligned} 2\vec{a} + 7\vec{b} - 5\vec{z} &= 2(2\vec{j} + \vec{k}) + 7(-3\vec{i} + 5\vec{j} + 4\vec{k}) - 5(\vec{i} - 3\vec{j} - \vec{k}) \\ &= (-21 - 5)\vec{i} + (4 + 35 + 15)\vec{j} + (2 + 28 + 5)\vec{k} \\ &= -26\vec{i} + 54\vec{j} + 35\vec{k} \end{aligned}$$

13.1/16



$$\begin{aligned} \vec{v} &= 8\cos(-40^\circ)\vec{i} + 8\sin(-40^\circ)\vec{j} \\ &= 8\cos(40^\circ)\vec{i} - 8\sin(40^\circ)\vec{j} \\ &\approx 6.128\vec{i} - 5.142\vec{j} \end{aligned}$$

13.1/20

$$\vec{u} = \vec{i} + \vec{j} + 2\vec{k}, \quad \vec{v} = -\vec{i} + 2\vec{k}$$

13.1/22

A unit vector in the same direction is $\frac{1}{\sqrt{6}}(\vec{i} - \vec{j} + 2\vec{k})$,
so a vector with length 2 in the same direction is

$$\frac{2}{\sqrt{6}}(\vec{i} - \vec{j} + 2\vec{k}).$$

13.1/24

$$\vec{v} = 3\vec{i} + y\vec{j}, \quad \|\vec{v}\| = \sqrt{9 + y^2} = 5 \Rightarrow y^2 = 16 \Rightarrow y = 4 \text{ or } y = -4$$

so there are two possible vectors, $3\vec{i} + 4\vec{j}$ and $3\vec{i} - 4\vec{j}$.

13.2/8



$$10\cos(-\frac{\pi}{4})\vec{i} + 10\sin(-\frac{\pi}{4})\vec{j} = 5\sqrt{2}\vec{i} - 5\sqrt{2}\vec{j}$$

13.2/13

Let $\vec{v} = a\vec{i} + b\vec{j}$ be the velocity of the plane relative to the air, and let $\vec{w} = 60 \cos(-\frac{\pi}{4})\vec{i} + 60 \sin(-\frac{\pi}{4})\vec{j} = 30\sqrt{2}\vec{i} - 30\sqrt{2}\vec{j}$ be the velocity of the air relative to the ground. Then the velocity of plane relative to the ground is $\vec{v} + \vec{w} = (a + 30\sqrt{2})\vec{i} + (b - 30\sqrt{2})\vec{j}$.

For the plane to end up heading due east, we must have $b - 30\sqrt{2} = 0 \Rightarrow b = 30\sqrt{2}$.

$$\text{so } \vec{v} = a\vec{i} + 30\sqrt{2}\vec{j}.$$

We are told that the airspeed is 500, which means $\|\vec{v}\| = 500$.

$$\text{thus } \sqrt{a^2 + 1800} = 500 \Rightarrow a = \sqrt{(500)^2 - 1800} = 10\sqrt{2482} \approx 498.2$$

$$\text{so } \vec{v} = 10\sqrt{2482}\vec{i} + 30\sqrt{2}\vec{j} \approx 498.2\vec{i} + 42.43\vec{j}$$

The angle of this vector is

$$\theta = \arctan\left(\frac{30\sqrt{2}}{10\sqrt{2482}}\right) \approx 0.08495 \text{ radians} \approx 4.868 \text{ degrees (north of east)}$$

The speed relative to the ground is

$$\|\vec{v} + \vec{w}\| = a + 30\sqrt{2} = 10\sqrt{2482} + 30\sqrt{2} \approx 540.6 \text{ km/hr}$$

13.2/14

Let \vec{v} be the velocity of the plane relative to the air.

We know $\|\vec{v}\| = 480$ km/hr, and $\vec{v} = x\vec{i} + 80\vec{k}$ because the plane is heading due east. Then $\sqrt{x^2 + 80^2} = 480 \Rightarrow x = \sqrt{480^2 - 80^2} = \sqrt{224000} \approx 473.3$

The velocity of the wind relative to the ground is

$$\vec{w} = 50\sqrt{2}\vec{i} + 50\sqrt{2}\vec{j} \approx 70.71\vec{i} + 70.71\vec{j}.$$

so the velocity of the plane relative to the ground is

$$\begin{aligned}\vec{v} + \vec{w} &= (473.3 + 70.71)\vec{i} + 70.71\vec{j} + 80\vec{k} \\ &= 544.0\vec{i} + 70.71\vec{j} + 80\vec{k}\end{aligned}$$

The ground speed is the magnitude of this vector:

$$\sqrt{544.0^2 + 70.71^2 + 80^2} = \boxed{554.4 \text{ km/hr}}$$

(Note: If one interprets "ground speed" to mean just the horizontal component of the velocity, we obtain

$$\sqrt{544.0^2 + 70.71^2} = 548.6 \text{ km/hr.}$$

13.2/23

The force exerted on the object from the first rope $\vec{F}_1 = 100 \cos(30^\circ)\vec{i} + 100 \sin(30^\circ)\vec{j} = 86.60\vec{i} + 50\vec{j}$ and the force exerted from the second rope is $\vec{F}_2 = 70 \cos(80^\circ)\vec{i} - 70 \sin(80^\circ)\vec{j} = 12.16\vec{i} - 68.94\vec{j}$. The sum of these two forces is $\vec{F}_1 + \vec{F}_2 = 98.76\vec{i} - 18.94\vec{j}$. See Figure 13.14. In order for the object to move vertically, the total force on the object must be in the form $\vec{F} = 0\vec{i} + 0\vec{j} + b\vec{k}$ for some b . Thus the force vector for the crane is

$$\vec{F}_c = -98.76\vec{i} + 18.94\vec{j} + b\vec{k}$$

for some b . To find b , we use the fact that $\|\vec{F}_c\| = 3000$. Thus,

$$\begin{aligned} \|\vec{F}_c\| &= 3000 \\ \sqrt{(98.76)^2 + (18.94)^2 + b^2} &= 3000 \\ b &= \pm 2998.31 \end{aligned}$$

We use the positive value of b since we want the object to go up rather than down. The force exerted by the crane is

$$\vec{F}_c = -98.76\vec{i} + 18.94\vec{j} + 2998.31\vec{k}$$

The total force acting on the object is $2998.31\vec{k}$, or 2998.31 newtons straight up.

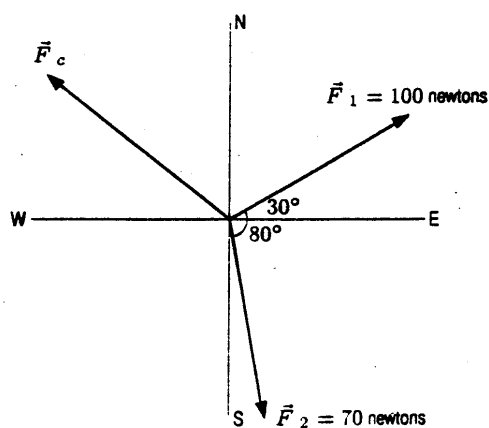


Figure 13.14: Horizontal forces on object