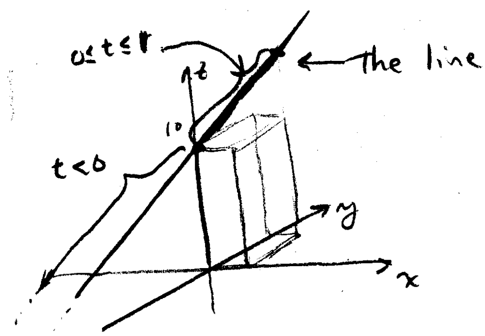


Not Assigned

(a) For $t<0$, we get the part of the line where $z<10$.

(b) For $0 \le t \le 1$, we get the line segment from $(0,0,10)$ to $(1,2,13)$.

17.1/18

A circle of radius 5 in the yz-plane centered at the origin is $\vec{r} = 5\cos t\,\vec{j} + 5\sin t\,\vec{k}$.

To center it at $(-1,0,-2)$, add $-i-2k$:

$$\boxed{\vec{r} = -\vec{i} + (5\cos t)\vec{j} + (5\sin t - 2)\vec{k}}$$

or $\boxed{x=-1,\ y = 5\cos t,\ z = 5\sin t - 2}$

17.1/22

The curve is parallel to the xy-plane through $(0,4,4)$, so $z=4$. Thus we have

$$\begin{Bmatrix} x = x \\ y = 4-5x^4 \\ z = 4 \end{Bmatrix}$$

or, parameterized by t instead of x,

$$\begin{Bmatrix} x = t \\ y = 4-5t^4 \\ z = 4 \end{Bmatrix}$$

$\boxed{17.1/10}$

The line contains the points $A=(3,-2,2)$ and $B=(0,2,0)$, so a vector parallel to the line is $\vec{AB} = -3\vec{i}+4\vec{j}-2\vec{k}$.

Then the line is

$$\vec{r} = 3\vec{i} - 2\vec{j} + 2\vec{k} + t(-3\vec{i}+4\vec{j}-2\vec{k})$$

or $\quad x = 3-3t, \quad y = -2+4t, \quad z = 2-2t$

(other answers are possible.)

$\boxed{17.1/30}$

The direction of l_1 is $2\vec{i} - \vec{j} + 3\vec{k}$, and the direction of l_2 is $-3\vec{i}+5\vec{j}$. These vectors are not parallel, so the lines are <u>not</u> parallel.

If the lines intersect, there must be numbers t_1 and t_2 such that

$$5+2t_1 = 4-3t_2, \quad -t_1 = 1+5t_2 \quad \text{and} \quad -2+3t_1 = 4$$

The last equation implies $t_1 = 2$. Then the first equation gives $9 = 4-3t_2 \Rightarrow t_2 = -\frac{5}{3}$. Putting t_1 and t_2 into the second equation gives

$$-2 = -\frac{22}{3}, \quad \text{which is false,}$$

so the lines do <u>not</u> intersect.

$\boxed{17.1/53}$ It is a straight line through the point $(3,5,7)$, parallel to the vector $\vec{v} = \vec{i} - \vec{j} + 2\vec{k}$. The motion slows down as it approaches $(3,5,7)$, momentarily stops at $(3,5,7)$, and then it speeds up as it travels along the line away from $(3,5,7)$.

$\boxed{17.1/46}$

The planes are

① $x+y+z=3$

and

② $x-y+2z=2$

From ① we have $z=3-x-y$; put this into ② to obtain

$$x-y+2(3-x-y)=2$$

$$-x-3y=-4$$

$$x=4-3y$$

Then

$$z=3-(4-3y)-y$$

$$=2y-1$$

Thus we have

$$x=4-3y$$

$$y=y$$

$$z=2y-1,$$

or, parameterized by t instead of y,

$$\boxed{\begin{aligned} x&=4-3t\\ y&=t\\ z&=2t-1\end{aligned}}$$

In vector form, $\boxed{\vec{r}=4\vec{i}-\vec{k}+t(-3\vec{i}+\vec{j}+2\vec{k})}$

(Other parameterizations are possible.)

$\boxed{17.1/48}$

Let $f(x,y) = x^2 + y^2$. Then $f_x(x,y) = 2x$, $f_y(x,y) = 2y$, and the

and the plane tangent to the graph of f at $(1,2,5)$ is

$$z = 5 + (2)(x-1) + (4)(y-2) = 2x + 4y - 5, \text{ or}$$

$$2x + 4y - z - 5 = 0$$

A vector normal to the tangent plane, and therefore normal (ie perpendicular) to the surface is

$$\vec{n} = 2\vec{i} + 4\vec{j} - \vec{k}.$$

So a line through $(1,2,5)$ that is perpendicular to the surface is

$$\vec{r} = \vec{i} + 2\vec{j} + 5\vec{k} + t(2\vec{i} + 4\vec{j} - \vec{k})$$

or

$$x = 1 + 2t,$$
$$y = 2 + 4t,$$
$$z = 5 - t.$$

$\boxed{17.1/52}$

(a) The paths are straight lines. The direction vector of the first is $\vec{i} - \vec{j} + 2\vec{k}$, and the direction vector of the second is $2\vec{i} + 2\vec{j} + \vec{k}$.

(b) To <u>collide</u>, the particles must be at the same point at the same time.

$\Rightarrow$ ① $-1 + t = -7 + 2t$, ② $4 - t = -6 + 2t$, and ③ $-1 + 2t = -1 + t$

The first equation implies $t = 5$, but then the second says $-1 = 4$; this means there is no solution, so the particles <u>do not collide</u>.

(c) The paths cross if there is a point that the first particle crosses at time t_1 and the second particle crosses at time t_2.

$\Rightarrow$ ① $-1 + t_1 = -7 + 2t_2$, ② $4 - t_1 = -6 + 2t_2$, ③ $-1 + 2t_1 = -1 + t_2$

The first equation gives $t_1 = -6 + 2t_2$. Then ② gives $10 - 2t_2 = -6 + 2t_2 \Rightarrow t_2 = 4$ so $t_1 = 2$. We must check that ③ is also satisfied: $-1 + (2)(2) = -1 + 4 \Rightarrow 3 = 3$ ✓.

So <u>the paths cross at $(1,2,3)$.</u>

$\boxed{17.2/7}$ $\quad x = 3\sin(t^2) - 1, \quad y = 3\cos(t^2)$

$$\frac{dx}{dt} = 3\cos(t^2)(2t) \qquad \frac{dy}{dt} = -3\sin(t^2)(2t)$$

$$= 6t\cos(t^2) \qquad\qquad = -6t\sin(t^2)$$

so $\vec{V}(t) = 6t\cos(t^2)\,\vec{i} - 6t\sin(t^2)\,\vec{j}$

$$\|\vec{V}(t)\| = \sqrt{[6t\cos(t^2)]^2 + [-6t\sin(t^2)]^2}$$

$$= \sqrt{36t^2\cos^2(t^2) + 36t^2\sin^2(t^2)}$$

$$= 6|t|\sqrt{\underbrace{\cancel{\cos^2(t^2)+\sin^2(t^2)}}_{1}}$$

$$= 6|t|$$

$\|\vec{V}(0)\| = 0$, so the particle stops at $\underline{t = 0}$.

$\boxed{17.2/4}$ $\quad x = 3\cos(t^2) \qquad y = 3\sin(t^2) \qquad z = t^2$

$$\frac{dx}{dt} = -6t\sin(t^2) \qquad \frac{dy}{dt} = 6t\cos(t^2) \qquad \frac{dz}{dt} = 2t$$

$$\frac{d^2x}{dt^2} = (-6t)(2t\cos(t^2)) - 6\sin(t^2) = -6\left(2t^2\cos(t^2) + \sin(t^2)\right)$$

$$\frac{d^2y}{dt^2} = (6t)(-2t\sin(t^2)) + 6\cos(t^2) = 6\left(-2t^2\sin(t^2) + \cos(t^2)\right)$$

$\frac{d^2z}{dt^2} = 2$, so

$$\boxed{\begin{aligned} \vec{V}(t) &= [-6t\sin(t^2)]\,\vec{i} + [6t\cos(t^2)]\,\vec{j} + 2t\,\vec{k} \\ \vec{a}(t) &= -6\left(2t^2\cos(t^2) + \sin(t^2)\right)\vec{i} + 6\left(-2t^2\sin(t^2) + \cos(t^2)\right)\vec{j} + 2\vec{k} \end{aligned}}$$

$\boxed{17.2/24}$

(a) The particle is traveling in the direction

$$\vec{PQ} = 2\vec{i} + 5\vec{j} + 3\vec{k}.$$

The (constant) velocity vector of the particle is a vector in the same direction as $\vec{PQ}$, but with magnitude 5. Thus

$$\vec{v} = 5\frac{\vec{PQ}}{\|\vec{PQ}\|} = \frac{5}{\sqrt{38}}(2\vec{i} + 5\vec{j} + 3\vec{k})$$

(b) In vector form, we have

$$\vec{r} = 3\vec{i} + 2\vec{j} - 5\vec{k} + \frac{5}{\sqrt{38}}t(2\vec{i} + 5\vec{j} + 3\vec{k});$$

the scalar components are

$$x = 3 + \frac{10}{\sqrt{38}}t, \quad y = 2 + \frac{25}{\sqrt{38}}t, \quad z = -5 + \frac{15}{\sqrt{38}}t$$

$\boxed{17.2/23}$ If we try $\vec{r} = 5\vec{i} + 4\vec{j} - 2\vec{k} + t(2\vec{i} - 3\vec{j} + \vec{k})$, we get the correct line, but the particle passes through P at $t=0$ - the wrong time. Introduce a translation in t to fix this:

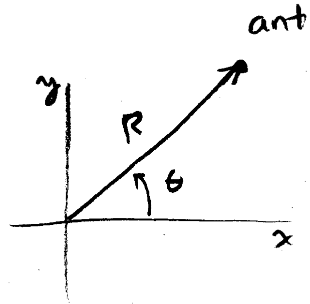
$$\vec{r} = 5\vec{i} + 4\vec{j} - 2\vec{k} + (t-4)(2\vec{i} - 3\vec{j} + \vec{k})$$

or $$\vec{r} = -3\vec{i} + 16\vec{j} - 6\vec{k} + t(2\vec{i} - 3\vec{j} + \vec{k})$$

Scalar equations are

$$x = 5 + 2(t-4) \qquad\qquad x = -3 + 2t$$
$$y = 4 - 3(t-4) \qquad \text{or} \qquad y = 16 - 3t$$
$$z = -2 + (t-4) \qquad\qquad z = -6 + t$$

$\boxed{17.2/33}$



$R(t) = t$ (assuming the ant starts at the center)

$\theta(t) = 2\pi t$ (assuming $\theta(0) = 0$)
(note: 1 revolution $= 2\pi$ radians)

(a) The position of the ant is

$$\vec{r}(t) = R(t)\cos(\theta(t))\,\vec{i} + R(t)\sin(\theta(t))\,\vec{j}$$
$$= t\cos(2\pi t)\,\vec{i} + t\sin(2\pi t)\,\vec{j}$$

(b) $$\vec{v}(t) = \left[-2\pi t\sin(2\pi t) + \cos(2\pi t)\right]\vec{i} + \left[2\pi t\cos(2\pi t) + \sin(2\pi t)\right]\vec{j}$$

The speed is

$$\|\vec{v}(t)\| = \left(\left[-2\pi t\sin(2\pi t) + \cos(2\pi t)\right]^2 + \left[2\pi t\cos(2\pi t) + \sin(2\pi t)\right]^2\right)^{\frac{1}{2}}$$
$$= \left(4\pi^2t^2\sin^2(2\pi t) - \cancel{2\pi t\sin(2\pi t)\cos(2\pi t)} + \cos^2(2\pi t) + 4\pi^2t^2\cos^2(2\pi t) + \cancel{2\pi t\sin(2\pi t)\cos(2\pi t)} + \sin^2(2\pi t)\right)^{\frac{1}{2}}$$
$$= \sqrt{4\pi^2t^2 + 1}$$

(c) $$\vec{a}(t) = \left[-4\pi^2t\cos(2\pi t) - 2\pi\sin(2\pi t) - 2\pi\sin(2\pi t)\right]\vec{i} + \left[-4\pi^2t\sin(2\pi t) + 2\pi\cos(2\pi t) + 2\pi\cos(2\pi t)\right]\vec{j}$$
$$= -4\pi\left[2\pi t\cos(2\pi t) + \sin(2\pi t)\right]\vec{i} + 4\pi\left[-2\pi t\sin(2\pi t) + \cos(2\pi t)\right]\vec{j}$$

The magnitude of the acceleration is

$$\|\vec{a}(t)\| = \ldots = 4\pi\sqrt{1+\pi^2t^2}$$