Math 113 – Calculus III **Exam 2 Practice Problems** **Spring 2003**

1. Suppose $\vec{u}$ is a unit vector, and $\vec{v}$ and $\vec{w}$ are two more vectors that are not necessarily unit vectors. Simplify the following expression as much as possible:

$$((\vec{v} \cdot \vec{u})\vec{u}) \cdot (\vec{v} \times \vec{w}) - (\vec{w} \times \vec{v}) \cdot (\vec{v} - (\vec{u} \cdot \vec{v})\vec{u}).$$

2. Let $P = (1, 1, 1)$, $Q = (1, -3, 0)$ and $R = (2, 2, 2)$.

   (a) Find the equation of the plane that contains the points P, Q, and R.

   (b) Find the area of the triangle formed by the three points.

   (c) Find the distance from the plane found in (a) to the point $(3, 4, 5)$.

3. We say two planes are perpendicular if their normal vectors are perpendicular. Given the following two planes (which are *not* perpendicular):

$$x + 2y + 4z = 1, \qquad -x + y - 2z = 5,$$

   find the equation of a plane that is perpendicular to *both* of these planes, and that contains the point $(3, 2, 1)$.

4. Let

$$g(x, y, z) = e^{-(x+y)^2} + z^2(x + y).$$

   (a) What is the instantaneous rate of change of g at the point $(2, -2, 1)$ in the direction of the origin?

   (b) Suppose that a piece of fruit is sitting on a table in a room, and at each point (x, y, z) in the space within the room, $g(x, y, z)$ gives the strength of the odor of the fruit. Furthermore, suppose that a certain bug always flies in the direction in which the fruit odor increases fastest. Suppose also that the bug always flies with a *speed* of 2 feet/second.

   What is the velocity vector of the bug when it is at the position $(2, -2, 1)$?

5. The path of a particle in space is given by the functions $x(t) = 2t$, $y(t) = \cos(t)$, and $z(t) = \sin(t)$. Suppose the temperature in this space is given by a function $H(x, y, z)$.

   (a) Find $\frac{dH}{dt}$, the rate of change of the temperature at the particle's position. (Since the actual function $H(x, y, z)$ is not given, your answer will be in terms of derivatives of H.)

   (b) Suppose we know that at all points, $\frac{\partial H}{\partial x} > 0$, $\frac{\partial H}{\partial y} < 0$ and $\frac{\partial H}{\partial z} > 0$. At $t = 0$, is $\frac{dH}{dt}$ positive, zero, or negative?

6. Let

$$f(x, y) = x^3 - xy + \cos(\pi(x + y)).$$

(a) Find a vector normal to the level curve $f(x, y) = 1$ at the point where $x = 1$, $y = 1$.

(b) Find the equation of the line tangent to the level curve $f(x, y) = 1$ at the point where $x = 1$, $y = 1$.

(c) Find a vector normal to the graph $z = f(x, y)$ at the point $x = 1$, $y = 1$.

(d) Find the equation of the plane tangent to the graph $z = f(x, y)$ at the point $x = 1$, $y = 1$.

7. Let

$$f(x, y) = (x - y)^3 + 2xy + x^2 - y.$$

(a) Find the linear approximation $L(x, y)$ near the point $(1, 2)$.

(b) Find the quadratic approximation $Q(x, y)$ near the point $(1, 2)$.

8. For each of the following functions, determine the set of points where the function is *not* differentiable. *Briefly* explain how you know it is not differentiable; use a picture if it helps.

(You do not have to *prove* that it is not differentiable; just identify the set of points based on your understanding of what differentiable means.)

(a) $f(x, y) = \left|x^2 + y^2 - 1\right|$

(b) $f(x, y) = (x^2 + y^2)^{1/4}$

(c) $f(x, y) = e^{-x^2+y}$

(d) $f(x, y) = \dfrac{x^3 - xy + 1}{x^2 - y^2}$

9. Let $H(x, y) = x^2 - y^2 + xy$, and suppose that x and y are both functions that depend on t. Express $\dfrac{dH}{dt}$ in terms of x, y, $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

10. Suppose f is a differentiable function such that

$$f(1, 3) = 1, \quad f_x(1, 3) = 2, \quad f_y(1, 3) = 4,$$
$$f_{xx}(1, 3) = 2, \quad f_{xy}(1, 3) = -1, \quad \text{and} \quad f_{yy}(1, 3) = 4.$$

(a) Find $\text{grad} f(1, 3)$.

(b) Find a vector in the plane that is perpendicular to the contour line $f(x, y) = 1$ at the point $(1, 3)$.

(c) Find a vector that is perpendicular to the surface $z = f(x, y)$ (i.e. the graph of f) at the point $(1, 3, 1)$.

(d) At the point $(1, 3)$, what is the rate of change of f in the direction $\vec{i} + \vec{j}$?

(e) Use a quadratic approximation to estimate $f(1.2, 3.3)$.