Math 113 – Calculus III **Practice Exam 1** **Spring 2003**

Name: _____

1. Match each function to one of the given graphs, and to one of the given contour diagrams. The plots are given in a separate handout.

$f(x,y)$	Graph	Contour Diagram
$x^2 + \left(\frac{3y}{2}\right)^2$		
$x^2 - \frac{1}{2}y^2$		
$x + \frac{y}{2}$		
$-x + \frac{y}{2}$		
$\sin(\pi x)$		
xy^2		
$\sqrt{x^2 + \left(\frac{3y}{2}\right)^2}$		
x^2		

2. (a) Consider the function
$$f(x, y) = x^2 - y - xe^y - 1.$$
Find a function $g(x, y, z)$ such that the graph of f is the level surface $g(x, y, z) = 5$.
$g(x, y, z) =$_____

(b) For each of the following functions, determine if the level surface $g(x, y, z) = 0$ can be expressed as the graph of a function $f(x, y)$. If it is not possible, explain why not. If is it possible, find the function $f(x, y)$.

i. $g(x, y, z) = x^2 + x + y^4 + z(z - 1)$
ii. $g(x, y, z) = \sin(x - y + 2x)$
iii. $g(x, y, z) = 1 - e^{x^2 - y + z}$

3. Describe the level surfaces of the function $g(x, y, z) = x^2 + 4y^2 + z$.

4. Determine the points (if there are any) where the following functions are *not* continuous. Justify your answers.

(a) $f(x, y) = \dfrac{\sin(x + y)}{x - y}$

(b) $g(x, y) = \dfrac{1}{x^2 + y^2 + 1}$

(c) $h(x, y) = \begin{cases} \frac{xy}{x^2+y^2} & (x, y) \neq (0, 0 \\ 0 & (x, y) = (0, 0) \end{cases}$

5. A train is traveling northwest at 10 miles per hour. A person in the train walks at 2 miles per hour from a window on the left side to a window directly across the train on the right side. (*Left* and *right* refer to the sides relative to a person facing the front of the train.)

Assume that $\vec{i}$ points east, and $\vec{j}$ points north. Express your answers in terms of these unit vectors.

(a) What is the velocity vector of the train?

(b) What is the velocity vector of the person relative to the train?

(c) What is the velocity vector of the person relative to the ground?

(d) What is the speed of the person relative to the ground?

6. (a) TRUE or FALSE? For any vectors $\vec{v}$ and $\vec{w}$, $(\vec{v} + \vec{w}) \cdot (\vec{v} - \vec{w}) = \|\vec{v}\|^2 - \|\vec{w}\|^2$. (Briefly explain.)

(b) TRUE or FALSE? For any vectors $\vec{v}$ and $\vec{w}$, $\|\vec{v} + \vec{w}\| = \|\vec{v}\| + \|\vec{w}\|$. (Briefly explain.)

7. Let
$$\vec{v} = 2\vec{i} + a\vec{j} + a^2\vec{k} \qquad \text{and} \qquad \vec{w} = (2a-3)\vec{i} + \vec{j} + \vec{k}.$$
   (a) What is the cosine of the angle between $\vec{v}$ and $\vec{w}$ when $a = 0$?
   (b) For which values of a are the vectors perpendicular? (Hint: there is at least one such value, so if you find none, check your calculation again!)

8. Let
$$\vec{v} = 3\vec{i} + 2\vec{j} + \vec{k}, \quad \text{and} \quad \vec{w} = \vec{i} - \vec{j} + \vec{k}.$$
   Find the following:
   (a) $-2\vec{v} + \vec{w}$
   (b) $\vec{v} \cdot \vec{w}$
   (c) A unit vector $\vec{u}$ that is parallel to $\vec{w}$.
   (d) The projection of $\vec{v}$ on to $\vec{u}$, where $\vec{u}$ is the unit vector you just found in (c).

9. Given the plane
$$x + y + z = 1,$$
   find the point in the plane that is closest to the point $P = (3, 3, 2)$.