1. Suppose the integral of some function f over a region R in the plane is given in polar coordinates as

$$\int_0^3 \int_0^{\frac{\pi}{2}} r^2 \, d\theta dr.$$

   (a) Sketch the region of integration R in the xy plane.

   (b) Convert this integral to Cartesian coordinates.

   (c) Evaluate the integral. (You may use either polar or Cartesian coordinates.)

2. Let W be the solid region where $x \geq 0$, $y \geq 0$, $z \geq 0$, $z \leq x + y$, and $x^2 + y^2 \leq 4$. (In other words, W is bounded by the yz plane, the xz plane, the xy plane, and the surfaces $z = x + y$ and $x^2 + y^2 = 4$.)

   Let $f(x, y, z) = 1 + x + 2z$ be the density of the material in this region.

   Express the total mass of the material in W as a triple integral in

   (a) rectangular coordinates,

   (b) cylindrical coordinates.

   Your expressions should be complete enough that, in principle, they could be evaluated, but not evaluate the integrals!

3. The following triple iterated integral uses spherical coordinates.

$$\int_0^{\pi/2} \int_0^{\pi/4} \int_{1/\cos\phi}^{2/\cos\phi} \rho^2 \sin\phi \, d\rho d\phi d\theta$$

   Sketch (and describe) the region of integration.

Brief Solutions

1. (a) Description instead of a sketch: R is the quarter of a disk with radius 3 that is in the first quadrant.

   (b) Remember that in polar coordinates, $dA = rd\theta dr$, so one of the "r"s in the integrand "belongs to" dA. This means that the function f, expressed in polar coordinates, is r (not r^2). Then, in Cartesian coordinates, f is $\sqrt{x^2+y^2}$. In Cartesian coordinates, the integral becomes

   $$\int_0^3 \int_0^{\sqrt{9-x^2}} \sqrt{x^2+y^2}\, dy\, dx.$$

   (c) $\frac{9\pi}{2}$

2. (a) $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{x+y} (1+x+2z)\, dz\, dy\, dx$

   (b) $\int_0^2 \int_0^{\pi/2} \int_0^{r\cos\theta + r\sin\theta} (1+r\cos\theta+2z) r\, dz\, d\theta\, dr$

3. (To be provided later.)