Math 213 – Calculus III **Practice Exam 2** **Fall 2002**

1. Let
$$g(x, y, z) = e^{-(x+y)^2} + z^2(x + y).$$
   (a) What is the instantaneous rate of change of g at the point $(2, -2, 1)$ in the direction of the origin?
   (b) Suppose that a piece of fruit is sitting on a table in a room, and at each point (x, y, z) in the space within the room, $g(x, y, z)$ gives the strength of the odor of the fruit. Furthermore, suppose that a certain bug always flies in the direction in which the fruit odor increases fastest. Suppose also that the bug always flies with a *speed* of 2 feet/second.

   What is the velocity vector of the bug when it is at the position $(2, -2, 1)$?

2. The path of a particle in space is given by the functions $x(t) = 2t$, $y(t) = \cos(t)$, and $z(t) = \sin(t)$. Suppose the temperature in this space is given by a function $H(x, y, z)$.
   (a) Find $\frac{dH}{dt}$, the rate of change of the temperature at the particle's position. (Since the actual function $H(x, y, z)$ is not given, your answer will be in terms of derivatives of H.)
   (b) Suppose we know that at all points, $\frac{\partial H}{\partial x} > 0$, $\frac{\partial H}{\partial y} < 0$ and $\frac{\partial H}{\partial z} > 0$. At $t = 0$, is $\frac{dH}{dt}$ positive, zero, or negative?

3. Let
$$f(x, y) = x^3 - xy + \cos(\pi(x + y)).$$
   (a) Find a vector normal to the level curve $f(x, y) = 1$ at the point where $x = 1$, $y = 1$.
   (b) Find the equation of the line tangent to the level curve $f(x, y) = 1$ at the point where $x = 1$, $y = 1$.
   (c) Find a vector normal to the graph $z = f(x, y)$ at the point $x = 1$, $y = 1$.
   (d) Find the equation of the plane tangent to the graph $z = f(x, y)$ at the point $x = 1$, $y = 1$.

4. Let
$$f(x, y) = (x - y)^3 + 2xy + x^2 - y.$$
   (a) Find the linear approximation $L(x, y)$ near the point $(1, 2)$.
   (b) Find the quadratic approximation $Q(x, y)$ near the point $(1, 2)$.

5. For each of the following functions, determine the set of points where the function is *not* differentiable. *Briefly* explain how you know it is not differentiable; use a picture if it helps.

   (You do not have to *prove* that it is not differentiable; just identify the set of points based on your understanding of what differentiable means.)

   (a) $f(x, y) = \left|x^2 + y^2 - 1\right|$
   (b) $f(x, y) = (x^2 + y^2)^{1/4}$
   (c) $f(x, y) = e^{-x^2+y}$
   (d) $f(x, y) = \dfrac{x^3 - xy + 1}{x^2 - y^2}$

6. An assortment of TRUE/FALSE or short answer questions:

   (a) TRUE or FALSE: If f is differentiable at $(0, 0)$, then f is continuous at $(0, 0)$.
   (b) TRUE or FALSE: If f is a continuous function defined on the region $x^2 + y^2 \leq 9$, then f has a maximum value and a minimum value in this region.
   (c) TRUE or FALSE: If $f_x(0, 0)$ exists, and $f_y(0, 0)$ exists, then f is differentiable at $(0, 0)$.
   (d) TRUE or FALSE: If f is differentiable at $(0, 0)$, then the tangent plane to the graph of f at $(0, 0)$ is given by $z = f(0, 0) + f_x(0, 0)x + f_y(0, 0)y$.
   (e) Give an example of a function $f(x, y)$ for which $(0, 0)$ is a local minimum, but for which the second derivative test fails to determine this classification.
   (f) Give an example of a function $g(x, y)$ which is differentiable everywhere except along the line $y = x$.
   (g) Let $H(x, y) = x^2 - y^2 + xy$, and suppose that x and y are both functions that depend on t. Express $\dfrac{dH}{dt}$ in terms of x, y, $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

7. Suppose f is a differentiable function such that

$$f(1,3) = 1, \quad f_x(1,3) = 2, \quad f_y(1,3) = 4,$$

$$f_{xx}(1,3) = 2, \quad f_{xy}(1,3) = -1, \quad \text{and} \quad f_{yy}(1,3) = 4.$$

(a) Find $\operatorname{grad} f(1,3)$.

(b) Find a vector in the plane that is perpendicular to the contour line $f(x,y) = 1$ at the point $(1,3)$.

(c) Find a vector that is perpendicular to the surface $z = f(x,y)$ (i.e. the graph of f) at the point $(1,3,1)$.

(d) At the point $(1,3)$, what is the rate of change of f in the direction $\vec{i} + \vec{j}$?

(e) Use a quadratic approximation to estimate $f(1.2, 3.3)$.

8. For each of the following functions, find and classify the critical points.

(a) $f(x,y) = x^3 - x^2 + 2xy + 2y^2$

(b) $g(x,y) = \sqrt{(x-1)^2 + y^2}$

(c) $h(x,y) = e^{-x+y} + x + y^2$

(d) $r(x,y) = (x-y)(x+y)x$ (Hint: Consider the contour lines $r(x,y) = 0$.)