Math 307 Supplemental Notes: "Lies Across" and the Colorado Corollary

Figure 1 shows a simple example of two rectangular regions that "lie across" each other: $f(A)$ lies across B vertically, and $f(B)$ lies across A vertically. This implies that $f^2(A)$ lies across A, so by the "Colorado Corollary", f^2 has a fixed point in A. This implies that f has a period 2 point in A with an itinerary $\overline{AB}$. (We've only shown two pieces of the domain of f. f might, in fact, be defined for all $\mathbb{R}^2$, but that doesn't change that the fact that there must be a period 2 point in A.)

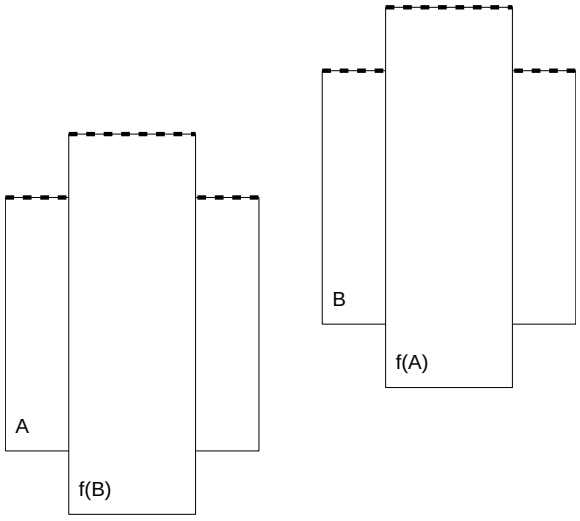

Figure 1: Simple example of two sets that "lie across" each other vertically.

Figure 2 shows why it is important for the "lies across" relations to all have the same orientation (i.e. vertically or horizontally) if we want to draw any useful conclusions. In this figure, $f(A)$ lies across B vertically, and $f(B)$ lies across A horizontally. In such a case, we can not conclude that $f^2(A)$ lies across A. Indeed, the figure shows an example where $f^2(A)$ is disjoint from A, so there are no period 2 points in A.

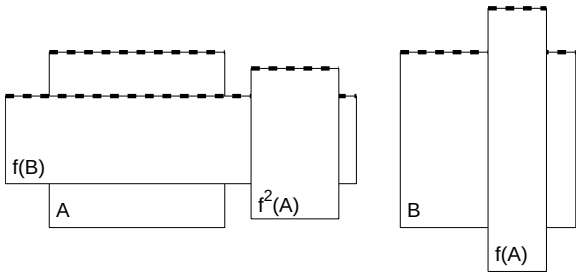

Figure 2: Example of two sets that lie across each other, but with different orientations.