Math 307 Supplemental Notes:
Sketching Phase Portraits for 2×2 Linear Systems

This is a brief summary of sketching a phase portrait for the linear system

$$\dot{\mathbf{x}} = A\mathbf{x},$$

where A is a 2×2 matrix.

- Find the eigenvalues of the matrix, and classify the equilibrium as a saddle, sink, source, spiral source, spiral sink, or center. (There are a few other special cases that we did not cover.)
- If the eigenvalues are real, find the associated eigenvectors, and draw the straight-line solutions.
- If the eigenvalues are real, distinct, and of the same sign, all trajectories except the straight-line solutons will approach the origin tangent to the eigenvector of the eigenvalue closest to zero.
- Draw the vector field along the x and y axes and the x and y nullclines. Note that at the points $(1, 0)$ and $(0, 1)$, the vector field is given by the first and second columns of A, respectively.
- Use the above information to sketch several representative trajectories.