Exercise T1.5.

Define the map f.

```
> f := x -> 2*x^2-5*x;
```

$$f := x \to 2\,x^2 - 5\,x$$

Find fixed points.

```
> solve(f(x)=x,x);
```

$$0,\ 3$$

Define a function for the second iterate of f.

```
> f2 := x -> f(f(x));
```

$$f2 := x \to \mathrm{f}(\mathrm{f}(x))$$

Note that f^2(x) is a degree 4 polynomial.

```
> f2(x);
```

$$2\,(2\,x^2 - 5\,x)^2 - 10\,x^2 + 25\,x$$

Find the fixed points of f2.

```
> solve(f2(x)=x,x);
```

$$0,\ 3,\ 1+\sqrt{2},\ 1-\sqrt{2}$$

We get four fixed points for f2, but two of them are also fixed points of f. The new points are 1 + sqrt(2) and 1-sqrt(2). These are the period 2 points.

Let's check:

```
> x0 := 1+sqrt(2);
```

$$x0 := 1+\sqrt{2}$$

```
> x1 := f(x0);
```

$$x1 := 2\,(1+\sqrt{2})^2 - 5 - 5\,\sqrt{2}$$

```
> x1 := simplify(f(x0));
```

$$x1 := 1-\sqrt{2}$$

x2 is f(f(x0)):

```
> x2 := simplify(f(x1));
```

$$x2 := 1+\sqrt{2}$$

We see that f(f(x0))=x0, x0 and x1 are period 2 points.

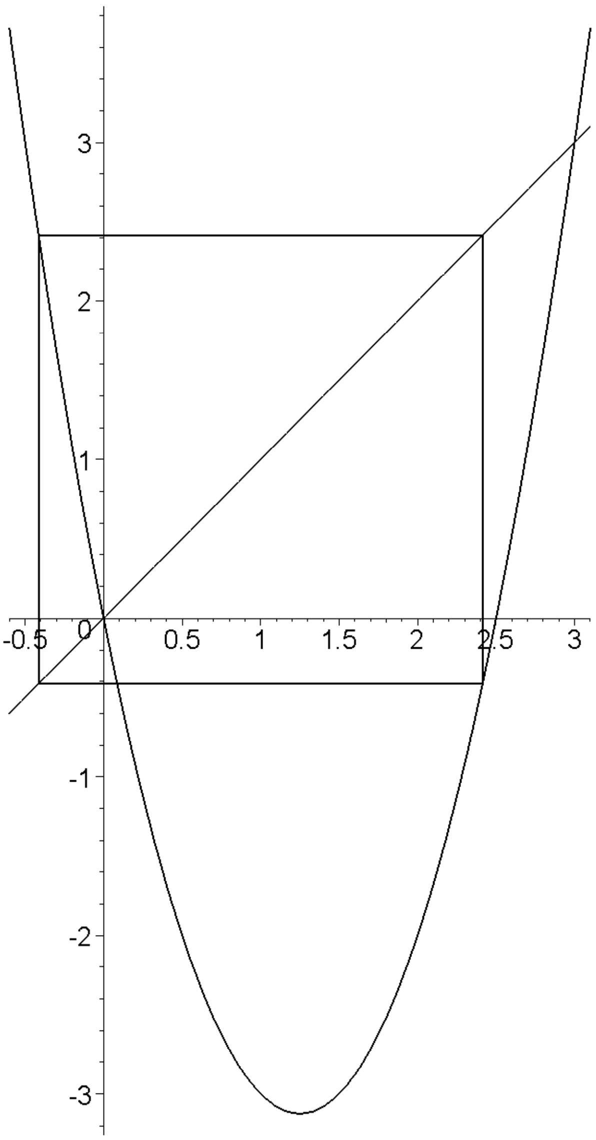
```
> id := x -> x;
```

$$id := x \to x$$

```
> p2data := [[x1,x2],[x2,x2],[x2,x1],[x1,x1],[x1,x2]];
```

$$p2data := [[1-\sqrt{2}, 1+\sqrt{2}], [1+\sqrt{2}, 1+\sqrt{2}], [1+\sqrt{2}, 1-\sqrt{2}], [1-\sqrt{2}, 1-\sqrt{2}], [1-\sqrt{2}, 1+\sqrt{2}]]$$

```
> plot({id,f,p2data},-0.6..3.1,scaling=constrained,color=black,thick
  ness=2);
```



The above plot includes the graph of f. The box shows the period-2 orbit.

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