General Solution to a Linear Second Order Differential Equation with Constant Coefficients

$$a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0, \qquad (a\neq 0) \tag{1}$$

General solution:

- If $b^2-4ac>0$, let $r_1=\frac{-b-\sqrt{b^2-4ac}}{2a}$, $r_2=\frac{-b+\sqrt{b^2-4ac}}{2a}$. The general solution is

$$y(x)=c_1e^{r_1x}+c_2e^{r_2x}$$

- If $b^2-4ac=0$, the general solution is

$$y(x)=c_1e^{-bx/(2a)}+c_2xe^{-bx/(2a)}$$

- If $b^2-4ac<0$, let $\omega=\frac{\sqrt{4ac-b^2}}{2a}$. The general solution is

$$y(x)=c_1e^{-bx/(2a)}\cos(\omega x)+c_2e^{-bx/(2a)}\sin(\omega x)$$