Math 312 - Applied Math: Social Sciences **Spring 2005**

Homework Assignment 2

Due *Friday, February 11*

1. Solve the following initial value problems.

   (a) $\dfrac{dy}{dt} = -y + e^{-t}, \quad y(0) = 1$

   (b) $\dfrac{dy}{dt} = 5y + t, \quad y(0) = 4$

2. In class, we saw that expressing the logistic equation

$$\frac{dp}{dt} = r\left(1 - \frac{p}{K}\right)p, \quad p(0) = p_0 \tag{1}$$

   with nondimensional variables resulted in the equation

$$\frac{dy}{d\tau} = (1-y)y, \quad y(0) = y_0 \tag{2}$$

   Solve this equation, and then convert your solution back to the dimensional variables to obtain the solution to (1).

   Hints: (i) The equation is separable. (ii) Dig out your Calculus II text or notes, and look up *partial fractions.*

3. Consider the first order differential equation

$$\frac{dp}{dt} = r\left(1 - \frac{p}{K}\right)p - ap,$$

   where $a \geq 0$ is a constant. This is a modification of the logistic equation.

   (a) If p represents the population of some animal, give a possible interpretation of the $-ap$ term. (Hint: Compare this problem to Problem 2 of Homework 1.)

   (b) Note that the dimensions of both r and a are $\mathcal{T}^{-1}$ (i.e. 1/time), so either of these parameters could be used to create a nondimensional time variable τ. Find the corresponding nondimensional differential equations that result for each choice. Be sure to express each equation using only a nondimensional parameter (or parameters) that you define in terms of the given dimensional parameters. How are the two nondimensional differential equations related?

   (c) Assume $p(0) = p_0 > 0$. How does the long term behavior of $p(t)$ depend on the parameters a and p_0? Are there positive values of a for which the population can become extinct?

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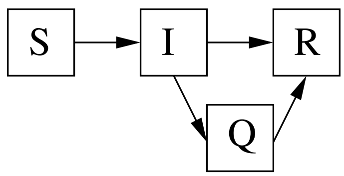
4. Consider the following modification of the SIR model of disease spreading.

$$\begin{aligned}\frac{dS}{dt} &= -rSI + \beta R \\ \frac{dI}{dt} &= rSI - \gamma I \\ \frac{dR}{dt} &= \gamma I - \beta R\end{aligned} \tag{3}$$

where $\beta > 0$ is a constant. Compare the differential equations to the SIR model, and explain the new behavior that has been included in these equations.

5. Suppose that during the spread of a disease, some of the infectives are placed in quarantine, so they do not interact with the susceptibles. Let Q be the size of the population in quarantine, and let I be the size of the population of infectives that are *not* in quarantine. The total population is then $N = S + I + Q + R$. The schematic for this SIQR model is



Suppose that the rate at which infectives are quarantined is proportional to I. The population in quarantine recovers at the same rate as the nonquarantined infectives.

Write down the system of four differential equations for this SIQR model.

6. Consider the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= -3x + y \\ \frac{dy}{dt} &= -y\end{aligned} \tag{4}$$

with the initial conditions $x(0) = x_0$ and $y(0) = y_0$. (Soon we will see a general method for solving a system like this.)

(a) Solve this initial value problem by solving the second equation for $y(t)$, and then putting $y(t)$ into the first equation and solving for $x(t)$.

(b) What are $\lim_{t\to\infty} x(t)$ and $\lim_{t\to\infty} y(t)$?