Homework Assignment 5

Due Friday, April 22

1. Let

$$P = \begin{bmatrix} 19/20 & 1/10 & 1/10 \\ 1/20 & 0 & 0 \\ 0 & 9/10 & 9/10 \end{bmatrix}$$
(1)

be the transition matrix of a Markov chain.

- (a) Draw the transition diagram that corresponds to this transition matrix.
- (b) Show that this Markov chain is regular.
- (c) Find the long-term probability distribution for the state of the Markov chain.
- 2. Consider the following transition diagram:



- (a) Find the transition matrix, and show that the Markov chain is regular.
- (b) Find the long term probability distribution of the states A, B, and C.
- 3. We consider a modification of the "Coin and Die" game. The rules for *Coin* are the same:
 - If the coin turns up **heads**, *Coin* wins.
 - If the coin turns up **tails**, it is *Die*'s turn.

The new rules for *Die* are:

- If the die turns up 1 or 2, *Die* wins.
- If the die turns up **3**, *Die* rolls again.
- If the die turns up 4, 5, or 6, it is *Coin*'s turn.
- (a) If *Coin* goes first, what is the probability that *Coin* wins, and what is the expected number of turns?
- (b) If *Die* goes first, what is the probability that *Coin* wins, and what is the expected number of turns?