Math 312 **Spring 2006**

Homework Assignment 1

Due *Friday, January 27.*

1. Find the following integrals.

   (a) $\displaystyle\int \frac{1}{x-a}\,dx$

   (b) $\displaystyle\int \frac{1}{x(1-x)}\,dx$ (Hint: Review *partial fractions* for integrating rational functions.)

   (c) $\displaystyle\int \frac{1}{x^b - x}\,dx, \quad b \neq 1$ (Hint: $\displaystyle\frac{1}{x^b-x} = \frac{1}{x^b(1-x^{1-b})} = \frac{x^{-b}}{1-x^{1-b}}$. Use a u-substitution.)

2. For each of these matrices, find the eigenvalues and eigenvectors. You must do these "by hand", but you may use a computer to check your answers.

   (a) $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 2 \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$

3. Before doing this problem, go to the computer lab in McGregory 201E, and try the MATLAB examples given in the web page

   `http://math.colgate.edu/~wweckesser/math312/MatlabNotes.html`

   Let
   $$A = \begin{bmatrix} 0 & 1.0 & 2.0 & 3.0 & 1.0 \\ 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 & 0 \\ 0 & 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0 & 0.75 & 0 \end{bmatrix}$$

   (a) Use MATLAB to find the eigenvalues of this matrix.

   (b) Use MATLAB to find an eigenvector of A associated with the eigenvalue that has the largest magnitude. (For a real number, the magnitude is simply the absolute value. For a complex number $z = a + ib$, the magnitude of z is $|z| = \sqrt{a^2 + b^2}$.)

4. Suppose we accept the following description as the basis for a model of the amount of a drug in a person's bloodstream:

   > When no additional drug is being added to the bloodstream, at each instant in time the amount of the drug decreases at a rate that is proportional to the amount at that time.

   (a) Convert the above statement into a mathematical equation that gives a continuous time model for the amount of the drug in the bloodstream. Define your variables and proportionality constants.

   (b) Suppose that the drug is being administered continuously through an intravenous (IV) line. The IV delivers m milligrams of the drug per minute (continuously). Give the new equation for this situation, and state the units of all your variables and constants.

5. Consider the following discrete time model for the population of an animal in a certain region. Because there is plenty of food in this region, each year the population increases by 8 percent when there is no hunting of the animal by humans. So, if p_n is the population in year n, we have

$$p_{n+1} = 1.08p_n.$$

For each of the following cases, give a new model that includes the effect of hunting on the population. (Assume that the hunting takes place during a short hunting season at the end of the year.)

(a) Each year, hunters kill 125 of the animals.

(b) Each year, hunters kill 5 percent of the animals.

(c) Each year, the number of animals killed by hunters is proportional to the product of the number of hunters and the number of animals. The number of hunters in year $n+1$ is proportional to the number of animals killed by hunters in year n. (You will have to introduce a second variable to represent the number of hunters in year n, along with some proportionality constants.)