Math 312 **Spring 2006**

Homework Assignment 2

Due *Friday, February 3.*

1. In some populations, if there are too few individuals, the population may die out. (Perhaps the animals normally travel in herds, and when the herd is too small, the individuals are easier targets for predators; or perhaps a small population ends up so spread out that it is harder for the animals to find mates during mating season.) Give a modification of the logistic equation that incorporates this phenomenon. That is, the model should have the following properties:

   - When the population is below a threshold level H, the population dies out.
   - When the population is between H and the carrying capacity K, the population increases.
   - When the population is greater than K, the population decreases.
   - A population of zero should be an equilibrium solution of the model.

   Include a plot of the right side of your equation as a function of p, and also include a plot (in one set of axes) of $p(t)$ versus t for several initial conditions.

2. The *Gompertz model* is another model of population growth. The differential equation is

$$\frac{dp}{dt} = c \ln\left(\frac{K}{p}\right) p$$

   where $c > 0$ and $K > 0$ are constants.

   (a) Plot the per capita growth rate (as a function of p), and explain the similarities and differences between this per capita growth rate and the per capita growth rate of the logistic equation.

   (b) Plot the right side of the differential equation (as a function of p).

   (c) Find the equilibrium solutions.

   (d) In one plot, make a rough sketch of several solutions, based on the plot in part (b). Include the equilibrium solutions.

   (e) Solve the differential equation explicitly. Use the initial condition $p(0) = P_0$.

   (f) Use your explicit formula to show that $\lim_{t\to\infty} p(t) = K$ if $P_0 > 0$.

3. Read the handout on Solow's Economic Growth Model. Do Exercises 4.1.2 and 4.1.4 from the handout.