

Homework 6Due *Friday, March 31*

1. Consider the nonlinear system

$$\begin{aligned}\frac{dx}{dt} &= x(1 - x - y) \\ \frac{dy}{dt} &= y(3 - 2x - y)\end{aligned}$$

In the following, do not assume x and y must be positive.

- (a) Find all the equilibrium points.
 - (b) Find the linearization at each equilibrium, classify the equilibrium point, and sketch the phase portrait for the linearized system.
 - (c) Sketch the phase portrait for the nonlinear system. Follow these steps:
 - i. Mark the locations of the equilibrium points.
 - ii. Sketch the nullclines (the curves where $\frac{dx}{dt} = 0$ or $\frac{dy}{dt} = 0$), and in the regions bounded by the nullclines, indicate the sign of $\frac{dx}{dt}$ and $\frac{dy}{dt}$.
 - iii. Near each equilibrium point, copy the the phase portait from part (b) to indicate the local behavior determined by the linear approximation.
 - (d) Suppose x and y represent two species that are competing for a common resource. If $x(0) > 0$ and $y(0) > 0$, what will happen to the populations of the species in the long term?
2. **Exercise 4.2.2** from the handout “Chapter 4: Linear Discrete Maps”.
3. **Exercise 4.3.3** from the handout “Chapter 4: Linear Discrete Maps”.