

# This is the Title of the Talk

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# Slow and Fast Subsystems

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$$\begin{aligned}\varepsilon \dot{x} &= x - \frac{1}{3}x^3 + y \\ \dot{y} &= -x + a \sin(2\pi\theta) \\ \dot{\theta} &= \omega\end{aligned}$$

# Slow and Fast Subsystems

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## Slow Subsystem (DAE)

$$\begin{aligned}y &= \frac{1}{3}x^3 - x \\ \dot{y} &= -x + a \sin(2\pi\theta) \\ \dot{\theta} &= \omega\end{aligned}$$

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↓ Eliminate  $y$  and desingularize

$$\begin{aligned}\dot{\theta} &= \omega(x^2 - 1) \\ \dot{x} &= -x + a \sin(\theta) \\ &(\text{Time reversed for } |x| < 1.)\end{aligned}$$

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$$t \rightarrow \varepsilon t$$

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## Critical Manifold

$$y = \frac{1}{3}x^3 - x$$