This is the Title of the Talk

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Slow and Fast Subsystems

$$\varepsilon \dot{x} = x - \frac{1}{3}x^3 + y$$
$$\dot{y} = -x + a\sin(2\pi\theta)$$
$$\dot{\theta} = \omega$$

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$$\downarrow \ \varepsilon = 0$$

Slow Subsystem (DAE)

$$y = \frac{1}{3}x^3 - x$$
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$\downarrow$ Eliminate y and desingularize

$$\dot{\theta} = \omega(x^2 - 1)$$
$$\dot{x} = -x + a\sin(\theta)$$

(Time reversed for $|x| < 1$.)

Slow and Fast Subsystems

$$\begin{aligned}\varepsilon\dot{x} &= x - \frac{1}{3}x^3 + y\\ \dot{y} &= -x + a\sin(2\pi\theta)\\ \dot{\theta} &= \omega\end{aligned}$$

$\xrightarrow{t \to \varepsilon t}$

$$\begin{aligned}\dot{x} &= x - \frac{1}{3}x^3 + y\\ \dot{y} &= \varepsilon(-x + a\sin(2\pi\theta))\\ \dot{\theta} &= \varepsilon\omega\end{aligned}$$

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Fast Subsystem

$$\begin{aligned}\dot{x} &= x - \frac{1}{3}x^3 + y\\ \dot{y} &= 0,\ \ \dot{\theta} = 0\end{aligned}$$

$\downarrow$ Eliminate y and desingularize

$$\begin{aligned}\dot{\theta} &= \omega(x^2 - 1)\\ \dot{x} &= -x + a\sin(\theta)\end{aligned}$$

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Slow and Fast Subsystems

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Slow Subsystem (DAE)

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Fast Subsystem

$$\dot{x} = x - \frac{1}{3}x^3 + y$$
$$\dot{y} = 0, \quad \dot{\theta} = 0$$

$\downarrow$ Eliminate y and desingularize

$$\dot{\theta} = \omega(x^2 - 1)$$
$$\dot{x} = -x + a\sin(\theta)$$

(Time reversed for $|x| < 1$.)

Critical Manifold

$$y = \tfrac{1}{3}x^3 - x$$