This is the Title
of the
Talk

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Slow and Fast Subsystems

$$\varepsilon\dot{x} = x - \frac{1}{3}x^3 + y$$
$$\dot{y} = -x + a\sin(2\pi\theta)$$
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$\downarrow \quad \varepsilon = 0$

Slow Subsystem (DAE)

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$\downarrow$ Eliminate y and desingularize

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(Time reversed for $|x| < 1$.)

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Critical Manifold

$$y = \frac{1}{3}x^3 - x$$

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