

Where

When

This is the Title of the Talk

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Slow and Fast Subsystems

$$\varepsilon \dot{x} = x - \frac{1}{3}x^3 + y$$

$$\dot{y} = -x + a \sin(2\pi\theta)$$

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$$\downarrow \quad \varepsilon = 0$$

Slow Subsystem (DAE)

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$$\downarrow \text{ Eliminate } y \text{ and desingularize}$$

$$\dot{\theta} = \omega(x^2 - 1)$$

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(Time reversed for $|x| < 1$.)

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$$t \rightarrow \varepsilon t$$

$$\begin{aligned}\dot{x} &= x - \frac{1}{3}x^3 + y \\ \dot{y} &= \varepsilon(-x + a \sin(2\pi\theta)) \\ \dot{\theta} &= \varepsilon\omega\end{aligned}$$

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Critical Manifold

$$y = \frac{1}{3}x^3 - x$$