

Chapter 2 Homework Answers

Math 316

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p. 54: 1-10, 15,16,18,23,24,29-31,38,41.

1. a) $S = \{(r, r), (r, g), (r, b), (g, r), (g, g), (g, b), (b, r), (b, g), (b, b)\}$

b) $S = \{(r, g), (r, b), (g, r), (g, b), (b, r), (b, g)\}$

2. $E_n = \{(x_1, x_2, \dots, x_{n-1}, 6), x_i \neq 6\}$ where n is the number of rolls until a 6 appears. The event $(\cup_{i=1}^{\infty} E_i)^c$ is the event that 6 never appears.

3. $E \cap F = \{(1, 2), (1, 4), (1, 6), (2, 1), (4, 1), (6, 1)\}$. $E \cup F$ occurs if the sum is odd or if at least one of the dice lands on 1. $F \cap G = \{(1, 4), (4, 1)\}$. $E \cap F^c$ is the event that neither of the dice lands on 1 and the sum is odd. $E \cap F \cap G = F \cap G$.

4. $A = \{1, 0001, 0000001, \dots\}$, $B = \{01, 00001, 00000001, \dots\}$,
 $(A \cup B)^c = \{000 \dots 000, 001, 000001, \dots\}$

5. a) $2^5 = 32$

b) $W = \{(1, 1, 1, 1, 1), (1, 1, 1, 1, 0), (1, 1, 1, 0, 1), (1, 1, 0, 1, 1), (1, 0, 1, 1, 1), (0, 1, 1, 1, 1), (0, 0, 1, 1, 1), (0, 1, 1, 1, 0), (1, 0, 1, 0, 1), (1, 1, 0, 0, 1), (1, 1, 0, 1, 0), (1, 1, 1, 0, 0), (1, 0, 1, 1, 0), (0, 0, 1, 1, 0), (1, 1, 0, 0, 0)\}$

c) 8

d) $A \cap W = \{(1, 1, 1, 0, 0), (1, 1, 0, 0, 0)\}$

6. a) $S = \{(1, g), (0, g), (1, f), (0, f), (1, s), (0, s)\}$

b) $A = \{(1, s), (0, s)\}$

c) $B = \{(0, g), (0, f), (0, s)\}$

d) $\{(1, s), (0, s), (1, g), (1, f)\}$

7. a) 6^{15} (b) $6^{15} - 3^{15}$ (c) 4^{15}

8. a) .8 (b) .3 (c) 0

9. Choose a customer at random. Let A be the event that this customer carries an American Express card. Let V be the event that the customer carries a VISA card. Then $P(A \cup V) = P(A) + P(V) - P(A \cap V) = .24 + .61 - .11 = .74$.

10. Let R and N denote the events that the student wears a ring or necklace, respectively.

a) $P(R \cup N) = 1 - .6 = .4$ (b) Since $P(R \cup N) = P(R) + P(N) - P(R \cap N)$ we see that $P(R \cap N) = .1$.

15. a) $\frac{4 \binom{13}{5}}{\binom{52}{5}}$ (b) $\frac{13 \binom{4}{2} \binom{12}{3} \binom{4}{1}^3}{\binom{52}{5}}$ (c) $\frac{\binom{13}{2} \binom{4}{2}^2 \binom{44}{1}}{\binom{52}{5}}$ (d) $\frac{13 \binom{4}{3} \binom{12}{2} \binom{4}{1}^2}{\binom{52}{5}}$ (e) $\frac{13 \binom{4}{4} \binom{48}{1}}{\binom{52}{5}}$

16. a) $\frac{6!}{6^5}$ (b) $\frac{6\binom{5}{2}5\cdot4\cdot3}{6^5}$ (c) $\frac{\binom{6}{2}4\binom{5}{2}\binom{3}{2}}{6^5}$ (d) $\frac{6\cdot5\cdot4\binom{5}{3}}{6^5}$ (e) $\frac{6\cdot5\binom{5}{3}}{6^5}$ (f) $\frac{6\cdot5\binom{5}{4}}{6^5}$ (g) $\frac{6}{6^5}$

18. $\frac{2\cdot4\cdot16}{52\cdot51}$

23. $P(\text{first higher}) + P(\text{second higher}) + P(\text{same}) = 1$. Since $P(\text{first higher}) = P(\text{second higher})$ and $P(\text{same}) = 1/6$, the answer is $5/12$

24. $P(X = 2) = P(X = 12) = 1/36$, $P(X = 3) = P(X = 11) = 2/36$, $P(X = 4) = P(X = 10) = 3/36$, $P(X = 5) = P(X = 9) = 4/36$, $P(X = 6) = P(X = 8) = 5/36$, $P(X = 7) = 6/36$.

26. Calculate the probability of winning craps.

Let $F_i \stackrel{\text{def}}{=} \text{the event that you win on the } i^{\text{th}} \text{ roll.}$

$P(F_1) = P(\text{roll } 7 \text{ or } 11) = \frac{6}{36} + \frac{2}{36} = \frac{2}{9}$

$P(F_2)$: Let r_1 be the first roll, r_2 the second roll. Then we need $r_1 \in \{4, 5, 6, 8, 9, 10\}$ and $r_1 = r_2$. Hence $P(F_2) = P(\text{two 4s}) + P(\text{two 5s}) + P(\text{two 6s}) + P(\text{two 8s}) + P(\text{two 9s}) + P(\text{two 10s}) = 2 \left(\left(\frac{3}{36}\right)^2 + \left(\frac{4}{36}\right)^2 + \left(\frac{5}{36}\right)^2 \right)$

$P(F_3)$: Now let r_3 be the third roll. Then we need $r_1 = r_3$ and $r_2 \neq r_1$ nor 7. (Below $\frac{27}{36}$ is the probability of not rolling a 4 nor a 7.) Hence $P(F_3) = 2 \left(\left(\frac{3}{36}\right)^2 \left(\frac{27}{36}\right) + \left(\frac{4}{36}\right)^2 \left(\frac{26}{36}\right) + \left(\frac{5}{36}\right)^2 \left(\frac{25}{36}\right) \right)$

In general, we see that $P(F_{i+1}) = 2 \left(\left(\frac{3}{36}\right)^2 \left(\frac{27}{36}\right)^{i-1} + \left(\frac{4}{36}\right)^2 \left(\frac{26}{36}\right)^{i-1} + \left(\frac{5}{36}\right)^2 \left(\frac{25}{36}\right)^{i-1} \right)$, for $i = 1, 2, \dots$

Since F_i and F_j are mutually exclusive for $i \neq j$, the desired probability is $\sum_{i=0}^{\infty} P(F_i) = \frac{2}{9} + 2 \sum_{i=0}^{\infty} \left(\frac{3}{36}\right)^2 \left(\frac{27}{36}\right)^i + 2 \sum_{i=0}^{\infty} \left(\frac{4}{36}\right)^2 \left(\frac{26}{36}\right)^i + 2 \sum_{i=0}^{\infty} \left(\frac{5}{36}\right)^2 \left(\frac{25}{36}\right)^i$.

Since $\sum_{i=0}^{\infty} r^i = \frac{1}{1-r}$ for $|r| < 1$, we see that this sums to $.49\overline{29}$.

29. a) $\frac{n(n-1)+m(m-1)}{(n+m)(n+m-1)}$ (b) $\frac{n^2+m^2}{(n+m)^2}$.

c) Putting all terms over the common denominator $(n+m)^2(n+m-1)$ we must show that

$$n^2(n+m-1) + m^2(n+m-1) \geq n(n-1)(n+m) + m(m-1)(n+m)$$

which is immediate upon multiplying through and simplifying.

30. a) $\frac{\binom{7}{3}\binom{8}{3}3!}{\binom{8}{4}\binom{9}{4}4!} = 1/18$ (b) $\frac{\binom{7}{3}\binom{8}{3}}{\binom{8}{4}\binom{9}{4}} - 1/18 = 1/6$ (c) $\frac{\binom{7}{3}\binom{8}{3} + \binom{7}{4}\binom{8}{3}}{\binom{8}{4}\binom{9}{4}} = 1/2$

31. $P(\text{complete}) = \frac{3!}{3^3} = 2/9$, $P(\text{same}) = \frac{3}{27} = 1/9$

38. $\frac{\binom{3}{2}}{\binom{n}{2}} = 1/2$. Hence, $n(n-1) = 12$ and $n = 4$.

41. $1 - \frac{5^4}{6^4}$.