Chapter 3 Homework Answers

Math 316
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p. 102: 1-10, 20, 21, 23, 28-30, 45, 47, 48, 52, 53, 57, 64.

1. Since we know $(1,1),(2,2),\dots(6,6)$ are not in the sample space. We only have 30 element in our reduced sample space. Since 10 of these are successes, the answer is 1/3.

2. E:= First lands on 6, F_i:= Sum is i, for $i = 2,3,\dots,12$: $P(E|F_j) = 0$ for $j = 2,3,4,5,6$;

$P(E|F_7) = \frac{P(E\cap F_7)}{P(F_7)} = \frac{1/36}{6/36} = 1/6$; Others are similar: $P(E|F_k) = \frac{1}{13-k}$ for $k = 7,8,9,10,11,12$.

3. E:= East has 3 spades, F:= North/South have a total of 8 spades

$P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{\binom{13}{8}\binom{5}{3}\binom{39}{18}\binom{21}{10}/\binom{52}{39}}{\binom{26}{13}\binom{13}{8}\binom{39}{18}/\binom{52}{39}} = 39/115 = .339.$

4. E:= at least one 6, F_i:= sum is i, for $i = 2,3,\dots,12$: $P(E|F_j) = 0$ for $j = 2,3,4,5,6$;

$P(E|F_{11}) = P(E|F_{12}) = 1$; $P(E|F_7) = \frac{P(E\cap F_7)}{P(F_7)} = \frac{2/36}{6/26} = 2/6$; Others similar: $P(E|F_k) = \frac{2}{13-k}$ for $k = 8,9,10$.

5. $\frac{(6)(5)(9)(8)}{(15)(14)(13)(12)} = 6/91.$

6. E:= First and third are white, F:= Exactly 3 whites

With Replacement: $P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{(8/20)^2(2)(8/20)(4/20)}{(4)(8/20)^3(4/20)} = 1/2;$

Without Replacement: $P(E|F) = \frac{(2)\binom{8}{3}\binom{4}{1}}{(4)\binom{8}{3}\binom{4}{1}} = 1/2.$

7. E:= At least one male, F:= Sibling is female: $P(F|E) = \frac{P(E\cap F)}{P(E)} = \frac{(2/4)}{(3/4)} = 2/3.$

8. E:= Oldest is female, F:= Both female: $P(F|E) = \frac{P(F\cap E)}{P(E)} = \frac{P(F)}{P(E)} = \frac{(1/4)}{(1/2)} = 1/2.$

9. E:= A white, F:= Two whites, G:= B white, H:= C white

$P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{P(E\cap G\cap H^c)+P(E\cap G^c\cap H)}{P(E\cap G\cap H^c)+P(E\cap G^c\cap H)+P(E^c\cap G\cap H)} \overset{ind^t}{=} \frac{P(E)P(G)P(H^c)+P(E)P(G^c)P(H)}{P(E)P(G)P(H^c)+P(E)P(G^c)P(H)+P(E^c)P(G)P(H)}$

$= \frac{(1/3)(2/3)(3/4)+(1/3)(1/3)(1/4)}{(1/3)(2/3)(3/4)+(1/3)(1/3)(1/4)+(2/3)(2/3)(1/4)} = 7/11.$

10. E:= First is spade, F:= Second and third are spades: $P(E|F) = \frac{P(F|E)P(E)}{P(F)} = 3!\frac{\binom{13}{3}/\binom{52}{3}}{2!\binom{13}{2}\binom{50}{1}/\binom{52}{3}} = 11/50.$

This can also be done by reducing the sample space: There are 50 cards left, 11 of which are spades.

20. a) $P(F|CS) = \frac{P(F\cap CS)}{P(CS)} = \frac{.02}{.05} = .4.$ b) $P(CS|F) = \frac{P(F\cap CS)}{P(F)} = \frac{.02}{.52} = \frac{1}{26} \approx .0385.$

21. a) $\frac{212+36}{212+36+198+54} = \frac{248}{500} = .496$ b) $P(W > 25|H > 25) = \frac{H>25\cap W>25}{H>25} = \frac{54}{54+198} = \frac{54}{252} = \frac{3}{14} \approx .214$
c) $P(W > 25|H < 25) = \frac{P(W>25\cap H<25)}{P(H<25)} = \frac{36}{212+36} = \frac{9}{62} \approx .145.$

23. a) $P(II = W) = P(I = W)P(II = W|I = W) + P(I = R)P(II = W|I = R) = \frac{2}{6}\frac{2}{3} + \frac{4}{6}\frac{1}{3} = \frac{4}{9}.$
b) $P(I = W|II = W) = \frac{P(I=W)P(II=W|I=W)}{P(II=W)} = \frac{(2/6)(2/3)}{4/9} = \frac{1}{2}.$

28. F:= First ace occurs at spot 20, E:= Spot 21 is ace of spades, G:= Spot 21 is two of clubs

a) $P(E|F)$: F can occur with the ace of spades, making E impossible. Hence we need F to occur without the ace of spades. This has probability 3/4. In this case there is a 1/32 chance that the ace of spades with be next, since we *know* that the ace of spades is still in our hand. The answer is $(3/4)(1/32) = 3/128$.

b) $P(G|F)$: Given F, there are $\binom{32}{3}(3!)(48!)$ ways of arranging cards where the remaining aces have not been laid down. Of these, there are $\binom{31}{3}(3!)(47!)$ ways such that the two of clubs is the twenty-first card. Hence the probability is $\frac{\binom{31}{3}(3!)(47!)}{\binom{32}{3}(3!)(48!)} = 29/1536.$

29. N:=All new when chosen the second time; E:= Initially, 3 news chosen; F:=Initially, 2 news chosen; G:=Initially, 1 new chosen; H:=Initially, no new chosen

$P(N) = P(N|E)P(E) + P(N|F)P(F) + P(N|G)P(G) + P(N|H)P(H)$

$= \frac{\binom{6}{3}\binom{9}{3}}{\binom{15}{3}^2} + \frac{\binom{7}{3}\binom{6}{1}\binom{9}{2}}{\binom{15}{3}^2} + \frac{\binom{8}{3}\binom{6}{2}\binom{9}{1}}{\binom{15}{3}^2} + \frac{\binom{6}{3}\binom{9}{3}}{\binom{15}{3}^2} = 18480/207025 = .0893.$

30. E:= Marble is white, F:= Marble is black, G:= First box selected

a) $P(E) = P(E|G)P(G) + P(E|G^c)P(G^c) = (1/2)(1/2) + (2/3)(1/2) = 7/12.$

b) $P(G|E) = \frac{P(E|G)P(G)}{P(E)} = \frac{(1/2)(1/2)}{1-(7/12)} = 3/5.$

45. E_i:= i^{th} coin chosen ($i = 1, 2, \ldots, 10$), H:= Head

$P(E_5|H) = \frac{P(H|E_5)P(E_5)}{P(H)} = \frac{P(H|E_5)P(E_5)}{P(H|E_1)P(E_1)+P(H|E_2)P(E_2)+\ldots+P(H|E_{10})P(E_{10})} = \frac{(5/10)(1/10)}{(1/10)[(1/10)+(2/10)+\ldots+(10/10)]} = \frac{1}{11}.$

47. E_i:= Roll an i ($i = 1, 2, 3, 4, 5, 6$), F:= All white

a) $P(F) = P(F|E_1)P(E_1) + P(F|E_2)P(E_2) + \ldots + P(F|E_6)P(E_6) = (1/6)(P(F|E_1) + P(F|E_2) + \ldots + P(F|E_6))$

$= (1/6)(\frac{1}{3} + \frac{\binom{5}{2}}{\binom{15}{2}} + \frac{\binom{5}{3}}{\binom{15}{3}} + \frac{\binom{5}{4}}{\binom{15}{4}} + \frac{\binom{5}{5}}{\binom{15}{5}} + 0) = .0757$ b) $P(E_3|F) = \frac{P(F|E_3)P(E_3)}{P(F)} = \frac{\frac{\binom{5}{3}}{\binom{15}{3}}(\frac{1}{6})}{.0757} = .0484$

48. E:= Both Silver, F:= At least one silver: $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{(1/2)}{(1-(1/4))} = 2/3.$

52. A:= She's accepted, M:= Mail on monday, T:= Mail on tuesday, etc.

a) $P(M) = P(M|A)P(A) + P(M|A^c)P(A^c) = (.15)(.6) + (.05)(.4) = .11$

Using this same reasoning we have $P(T) = .16$, $P(W) = .19$, $P(Th) = .15$, and $P(F) = .14$. We will need these.

b) $P(T|M^c) = \frac{P(M^c|T)P(T)}{P(M^c)} = \frac{(1)(.16)}{1-.11} = 16/89.$

c) $P(A|M^c \cap T^c \cap W^c) = \frac{P(A \cap M^c \cap T^c \cap W^c)}{P(M^c \cap T^c \cap W^c)} \overset{Mult.\ Prin.}{=} \frac{P(A)(1-[P(M|A)+P(T|A)+P(W|A)])}{1-[P(M)+P(T)+P(W)]} = \frac{(.6)(1-[.15+.20+.25])}{1-[.11+.16+.19])} = \frac{12}{27}.$

d) $P(A|Th) = \frac{P(Th|A)P(A)}{P(Th)} = \frac{(.15)(.6)}{.15} = .6$

Note here that A and Th are independent events since $P(A|Th) = P(A)$ – not at all intuitive.

e) $P(A|no\ mail) = \frac{P(no\ mail|A)P(A)}{P(no\ mail)} = \frac{(1-[.15+.20+.25+.15+.1])(.6)}{1-(.11+.16+.19+.15+.14)} = 9/25.$

53. E:= Component 1 works, F:= System functions

$P(E|F) = \frac{P(F|E)P(E)}{P(F)} = \frac{(1)(1/2)}{1-(1/2)^n}.$

57. a) Two cases: up then down OR down then up, both have probability $p(1-p)$. Hence, the answer is $2p(1-p)$.

b) We need two ups and one down. There are $\binom{3}{1}$ choices for the down. Hence, the answer is $\binom{3}{1}p^2(1-p)$.

c) Our reduced sample space is (u,u,d), (u,d,u), (d,u,u), where u stands for up, and d for down. Of these, two are successes. Hence, the answer is 2/3.

64. With (a), our probability is obviously p. With (b), we may assume, without loss of generality, that the correct answer is T (true). The probability that they both answer T is p^2. If they disagree, they answer either TF or FT. In either case, the probability of disagreeing is $p(1-p)$. Hence, the probability that they get it correct in case (b) is $p^2 + (1/2)(p(1-p) + (1-p)p) = p$. Hence, either stategy has the same probability, p.