Section 14.7: Maxima and minima for $z(x,y)$

> with(plots):

Warning, the name changecoords has been redefined

Terminology: Consider $z(x,y)$ with $(a,b)$ in the domain of $z(x,y)$. We say:

- $z(x,y)$ has a local maximum (resp. minimum) at $(a,b)$ when $z(a,b) > z(x,y)$ (resp. $z(a,b) < z(x,y)$) on some disc containing $(a,b)$;

- $z(x,y)$ has an absolute maximum (resp. minimum) at $(a,b)$ when $z(a,b) > z(x,y)$ (resp. $z(a,b) < z(x,y)$) on the entire domain.

The function (surface) $z = \frac{\sin(x)\sin(y)}{xy}$ exhibits both local and absolute extrema on its domain of definition.

> plot3d((sin(x)*sin(y))/(x*y), x=-10..10, y=-10..10);

Example 2: Classify the extrema of $z = xy(1 - x - y)$ using the Second Derivative Test.

> plot3d(x*y*(1-x-y), x=-0.25..1.25, y=-0.25..1.25);

> contourplot(x*y*(1-x-y), x=-0.25..1.25, y=-0.25..1.25, contours=40, color=blue);
Example 3: Classify the extrema of $z = (5x^2 + 3y^2 + 1)\exp(-x^2 - y^2)$ with the aid of a surface and contour plot.

```plaintext
> p01:=plot([[0,0],[1/3,1/3],[0,1],[1,0]],style=point);
> p02:=contourplot(x*y*(1-x-y), x=-0.25..1.25, y=-0.25..1.25,contours=40,color=blue):
> display(p01,p02);
```

```plaintext
> plot3d((5*x^2 + 3*y^2 + 1)*exp(-x^2 - y^2), x=-3..3, y=-3..3);
```
\begin{verbatim}
> contourplot((5*x^2 + 3*y^2 + 1)*exp(-x^2 - y^2), x=-3..3, y=-3..3, contours=15, color=blue);

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\end{verbatim}