Section 1.7: Functions

**Definition 1.22** Let $A$ and $B$ be sets. A function from $A$ to $B$ is any assignment of objects in $A$ to objects in $B$ so that for each $a$ in $A$, the function assigns a unique object in $B$ to $a$.

Write $f : A \to B$ to denote a function from $A$ to $B$ and $f(a)$ to denote the unique object in $B$ that $f$ assigns to $a$.

If $A = \mathbb{N}$, then we say that $f$ is a number theoretic function.

**Definition 1.23** Let $f : A \to B$ be a function.

(i) We say $f$ is one-to-one if

\[ a_1 \neq a_2 \text{ in } A \text{ implies } f(a_1) \neq f(a_2) \text{ in } B. \]

(ii) We say $f$ is onto if for each $b$ in $B$, there is an $a$ in $A$ with $f(a) = b$.

**Definition 1.24** Let $f : \mathbb{N} \to B$ be a number theoretic function.

(i) We say $f$ is multiplicative if $f(mn) = f(m)f(n)$ whenever $(m,n) = 1$.

(ii) We say $f$ is completely multiplicative if $f(mn) = f(m)f(n)$ for any pair of natural numbers, $m$ and $n$. 
