Section 1.8: The number of positive divisors

**Definition 1.25** Let \( \nu : \mathbb{N} \to \mathbb{N} \) be the number theoretic function with \( \nu(n) \) equal to the number of positive divisors of \( n \). That is,

\[
\nu(n) = \sum_{1 \leq d \leq n \text{ and } d|n} 1.
\]

**Theorem 1.26** Let \( n > 1 \) and write \( n = p_1^{k_1} p_2^{k_2} \cdots p_r^{k_r} \). Then

\[
\nu(n) = \prod_{i=1,2,\ldots,r} (k_i + 1).
\]

**Corollary** The function \( \nu \) is multiplicative.

**Theorem 1.27** For \( n > 1 \),

\[
\frac{n^{\nu(n)}}{2} = \prod_{1 \leq d \leq n \text{ and } d|n} d.
\]