Part IV. Probability

Chapter 14

Computing a probability by counting

- Four tickets in a box labeled “1”, “2”, “3” and “4”. Tickets are drawn without replacement. What is the probability the second is “4”? 

- There are 12 possible ways to draw two tickets: 1,2; 1,3; 1,4; 2,1; 2,3; 2,4; 3,1; 3,2; 3,4; 4,1; 4,2; and 4,3. Of these, 3 have a “4” in the second position. So the probability is 3/12 = 1/4.

Computing probabilities by counting

- Which has the greater probability: rolling a “9” with three dice, or rolling a “10”? 

Both sums can be achieved in 6 ways:

9: 1 2 6; 1 3 5; 1 4 4; 2 3 4; 2 2 5; 3 3 3

10: 1 4 5; 1 3 6; 2 2 6; 2 3 5; 2 4 4; 3 3 4
Probabilities by counting [continued]

• But, considering these, we see there are 25 arrangements for the six configurations that give a “9” and 27 arrangements that give a “10.” So

\[ P(\text{sum} = 9) = \frac{25}{216} \text{ and} \]
\[ P(\text{sum} = 10) = \frac{27}{216} \]

Mutually exclusive events and the Addition Rule [aka the “Or” Rule]

• Two events are mutually exclusive if the occurrence of one event prevents the other.

• The Addition Rule [Version I]. If A and B are mutually exclusive events, then

\[ P(A \text{ or } B) = P(A) + P(B). \]

The Addition Rule [Version II]

• If the two events under consideration are not exclusive, then the sum will give an answer bigger than the probability of one or the other. But

• The Addition Rule [Version II] If A and B are any events, then

\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B). \]
<table>
<thead>
<tr>
<th>Independence and exclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>• An independent pair of event means the occurrence of one has no affect on the chances of the other. We are interested in independent events when faced with questions about one event and another. If the events are independent, we multiply the probabilities. If they are not, we must use a conditional probability when taking the product.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Independence and exclusion [continued]</th>
</tr>
</thead>
<tbody>
<tr>
<td>• To say a pair of events is exclusive means that the occurrence of one event completely determines [the impossibility of] the other. [This is the antithesis of independence.] We are interested in exclusion when asking about the probability of one event or another. If there is exclusivity, we add the probabilities. If not, the sum will be too big, and the true chance must be computed in different manner.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Independence and exclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>• As suggested on the last slide, independence and exclusion are two possible extreme situations. Pairs of events can fall somewhere in between these two extremes.</td>
</tr>
</tbody>
</table>
Paradox of Chevalier de Mere

• Game 1: Roll four dice. Win: At least one ‘1’ appears. P(win) = P('1' or '1' or '1' or '1') = (1/6) + (1/6) + (1/6) + (1/6) = 2/3.

• Game 2: 24 rolls of a pair of dice. Win: At least one pair of ‘1’s appears. P(win) = 24*(1/36) = 2/3.

• Both probability computations misuse the Or rule. The events in question are not mutually exclusive.

Paradox of Chevalier de Mere

• Correct computations for Game 1:
  
  • Method 1: P(win) = 2/3 - P('1' and '1' and '1' and '1') = (2/3) - (1/6) = 0.67 or 67%.
  
  • Method 2: P(win) = 1 - P(loss) = 1 - P('no 1' and 'no 1' and 'no 1' and 'no 1') = 1 - (5/6) = 0.52 = 52%.

  • Method 1 is incorrect. The Or rule does not extend easily beyond two events.