Part IV. Probability

Chapter 15: The Binomial Formula

Some motivating examples

• A coin is tossed four times. What is the chance of getting exactly 3 heads?

• A pair of dice is rolled five times. What are the chances of rolling exactly 2 sets of double sixes?

• A box contains tickets numbered 1 through 10 and seven draws are made with replacement. What is the chance that exactly 4 draws are odd numbered tickets?

Common features of these examples

• Each situation involves repeated trials.

• The number of trials is fixed in advance.

• The trials are independent events.

• The probability of the desired observation is the same from trial to trial.
Binomial coefficients

- Suppose you have two types of objects A and B. How many ways can you list n of these so that exactly k are of type A [so that (n - k) are of type B]?

  The solution is to compute \( \frac{n!}{k!(n-k)!} \)
  - In this formula, \( n! = n \times (n-1) \times \ldots \times 2 \times 1 \) and \( 0! = 1 \)
  - We will denote this computation by \((n,k)\).

The Binomial Formula

- Suppose event A occurs with fixed chance \( p \). The chance of event A occurring exactly \( k \) times in \( n \) repeated and independent trials is

  \[
  \binom{n}{k} p^k (1 - p)^{n-k}
  \]

Rule Recap

- The “Not” Rule. \( P(A) = 1 - P(\text{not } A) \)
- The “And” Rule. \( P(A \text{ and } B) = P(A) \cdot P(B \mid A) \).
  - If A and B are independent events, this rule becomes \( P(A \text{ and } B) = P(A) \cdot P(B) \).
- The “Or” Rule \([V2]\). \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \).
  - If A and B are mutually exclusive events, this becomes \( P(A \text{ or } B) = P(A) + P(B) \)
- The Binomial Formula. Suppose event A occurs with fixed chance \( p \). The chance of event A occurring exactly \( k \) times in \( n \) repeated and independent trials is \((n,k) \cdot p^k (1 - p)^{n-k}\)