Hypothesis testing

Chapter 26: One sample t-tests

One sample z-tests

- When testing for the significance of a single sample based on a z-test, we are assuming a normal distribution for all possible sample outcomes generated by a box model that corresponds to a particular null hypothesis. This assumption allows us to estimate the chance of seeing a particular outcome in a sample using normal approximations.

  - The idea: A small probability associated to a sample outcome is evidence against the null hypothesis
z-tests and t-tests

- The assumption that the possible sample outcomes are normally distributed is justified in situations in which the sample size is large.
  - Recall the Central Limit Theorem: For a large sample size n, the probability histogram for the possible outcomes will be more or less normal.

- Given this, we should be suspicious about using a z-test procedure when testing the significance of a small sample against a null hypothesis. To get around this problem we consider a slight modification of the z-test, which is known as a one-sample t-test.

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t-test: an example

Example [Cf Sec 26.6] Calibrating a new spectrophotometer.
- Manufactured test sample: 70 ppm of CO
- Assume: Measurement = 70 + measurement error and that the errors are normally distributed with an EV= 0 and an unknown SD.
  - We summarize this statement by saying that the measurements follow a Gaussian model
- The first twelve test sample measurements yield: 57.2, 71.7, 63.4, 73.9, 61.5, 60.7, 73.1, 59.4, 67.1, 76.5, 72.0, 60.2
- Do these suggest more than chance measurement error at work? That is could there be bias at work, so that
  Measurement = 70 + measurement error + bias
Null hypothesis: There is no bias. That is, the sample measures reflect only chance measurement errors [that are normally distributed with an average of 0].
  - Alternative hypothesis: There is a bias present. [The machine may be broken, or the test procedure invalid, or the test sample is not what it seems, or ... ]

Box model for NH: [ all possible sample measures ]
  - 12 draws
  - Ave of box = Ave (measures) = 70 + Ave (errors) = 70
  - SD of box = SD (measures) = SD+ (samples) by bootstrapping = 6.8 [to one decimal place]
A note on the last example

- NB. The analysis in the last example ‘changes’ only slightly if one focuses on sample measurement errors as opposed to the sample measurements. Indeed, you should get the same t-statistic from analyzing either variable since they are related by a change of scale through the Gaussian assumption

- Sample measurement = Actual value + [bias +] measurement error.