1. The curves \( y = kx^2 \) all approach \((0, 0)\) as \( x \to 0 \), so if the limit of \( f(x, y) = \frac{x^2}{x^2 + y} \) exists, it must be the same as the limit of \( f(x, kx^2) \) for every value of \( k \). But

\[
\lim_{x \to 0} \frac{x^2}{x^2 + (kx^2)} = \frac{1}{1 + k},
\]

so the limit is different for different values of \( k \); and hence the limit of \( f \) as \((x, y) \to (0, 0)\) does not exist.

3. (a)

(b) Yes, it appears that in any neighborhood of \((0, 0)\), the values of \( f(x, y) \) are close to 0, which is \( f(0, 0) \) by definition.

4. Figure 11.104 in the text is hard to interpret. Here is a better picture:
(a) Because $f(0,y)$ and $f(x,0)$ are the constant functions 0, they are clearly continuous.

(b) We saw in (a) that the rays in the $y$-axis, where $x = 0$, have this property. So take the ray $y = kx$ from the origin: On that ray,

$$f(x, kx) = \frac{x(kx)}{x^2 + (kx)^2} = \frac{k}{1 + k^2},$$

which is a constant (depending on $k$). Thus, the entire ray lies on the contour $f = k/(1 + k^2)$.

(c) No, because different paths to the origin approach different $z$-values, even along the straight-line paths as we saw in (b) — different values of $k$ yield different limits.