Problems 15.5, Page 246

2. \( \int_{\pi/2}^{3\pi/2} \int_{1}^{2} f(r, \theta) \, dr \, d\theta \)

3. \( \int_{0}^{2\pi} \int_{0}^{\sqrt{2}} f(r, \theta) \, dr \, d\theta \)

4. \( \int_{0}^{\pi/2} \int_{0}^{0.5} f(r, \theta) \, dr \, d\theta \)

6.

9.

11.

7.

12. With the substitution \( u = r^2 \) (so that \( du = 2r \, dr \) and, as \( r \) varies from 0 to 2, \( u \) varies from 0 to 4), we get

\[
\int_{0}^{2\pi} \int_{0}^{2} (\sin r^2) r \, dr \, d\theta = \int_{0}^{2\pi} \left[ \frac{1}{2} \cos u \right]_{0}^{4} \, d\theta = \int_{0}^{2\pi} \frac{1}{2} (1 - \cos 4) \, d\theta = \pi (1 - \cos 4) .
\]

15.

16.

The graphs for #15 and #16:
15. 
\[
\int_{\pi/2}^{3\pi/2} \int_0^1 (r \cos \theta) r \, dr \, d\theta = \left( \int_0^1 r^2 \, dr \right) \left( \int_{\pi/2}^{3\pi/2} \cos \theta \, d\theta \right) = \left( \left[ \frac{1}{3} r^3 \right]_0^1 \right) \left( [\sin \theta]_{\pi/2}^{3\pi/2} \right) = -\frac{2}{3}
\]

16. 
\[
\int_0^{\pi/4} \int_0^2 (r \cos \theta) (r \sin \theta) r \, dr \, d\theta = \left( \int_0^2 r^3 \, dr \right) \left( \int_0^{\pi/4} \cos \theta \, d\theta \right) = \left( \left[ \frac{1}{4} r^4 \right]_0^2 \right) \left( \left[ \frac{1}{2} \sin^2 \theta \right]_0^{\pi/4} \right) = 4 \left( \frac{1}{4} \right) = 1
\]

19. The two surfaces meet where \( \sqrt{8 - x^2 - y^2} = \sqrt{x^2 + y^2} \), or \( x^2 + y^2 = 4 \), the circle with radius 2 centered at \( x = 0 \), \( y = 0 \) (and \( z = 2 \)), so the solid exists over the disc of radius 2 centered at the origin in the \( xy \)-plane. So:
\[
\text{Volume} = \int_0^{2\pi} \int_0^2 (\sqrt{8 - r^2} - r) r \, dr \, d\theta = \left( \int_0^2 (\sqrt{8 - r^2} - r) r \, dr \right) \left( \int_0^{2\pi} d\theta \right) = \left( \left[ -\frac{1}{3} (8 - r^2)^{3/2} - \frac{1}{3} r^3 \right]_0^2 \right) \left( \left[ 2\theta + \sin \theta \right]_0^{2\pi} \right) = 2\pi \left( \sqrt{512} - 16 \right).
\]

21. (a)
\[
\text{Population} = \int_{\pi/2}^{3\pi/2} \int_0^1 \delta(r, \theta) r \, dr \, d\theta
\]

(b) We want \( \delta \) to decrease as \( r \) increases, so we want the factor of \( \delta \) with \( r \) in it to be \( 4 - r \); and as \( \theta \) moves from \( \pi/2 \) to \( \pi \) and then to \( 3\pi/2 \), we want \( \delta \) first to decrease and then to increase, so we want the factor of \( \delta \) with \( \theta \) in it to be \( 2 + \cos \theta \). So the best answer of these three is (i).

(c)
\[
\text{Population} = \int_{\pi/2}^{3\pi/2} \int_1^4 (4 - r)(2 + \cos \theta) r \, dr \, d\theta = \left( \int_1^4 (4 - r) r \, dr \right) \left( \int_{\pi/2}^{3\pi/2} (2 + \cos \theta) \, d\theta \right)
= \left( \left[ 2r^2 - \frac{1}{3} r^3 \right]_1^4 \right) \left( \left[ 2\theta + \sin \theta \right]_{\pi/2}^{3\pi/2} \right) = (30 - 21) (2\pi + (-1 - 1)) = 18(\pi - 1) \approx 38.5
\]